

Sensorless State-Space Control of Elastic Two-Inertia Drive System Using a Minimum State Order Observer

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Abstract

The paper presents sensorless state-space control of two-inertia drive system with resilient coupling. The control structure contains an I+PI controller for load speed regulation and a state feedback controller for effective vibration suppression of the elastic coupling. Mechanical state variable of two-inertia drive are obtained by using a linear minimum-order (Gopinath) state observer. The design of the combined (I+PI and state feedback) controller is achieved with the extended version of the modulus criterion [5]. The dynamic behavior of presented control structure has been examined, for different conditions, using MATLAB/SIMULINK simulation.

1. Introduction

Some electrically-driven mechanical systems, such as machine tools, robots, antennas, elevators and rolling mills have resilient couplings between the motor and controlled process. Elastic multi-inertia systems like these, which mostly are weakly damped, often can be approximated through an elastic two-mass system. In this case the classical PI control with motor speed feedback has poor performances and can not be applied for low values of stiffness constant.

Several control structures have been developed for suppression of torsion vibrations. These one combined the PI motor speed control with additional feedback from difference between motor and load speeds or from torsion torque or load speed [1–3], [8–10], [12]. Many of these control structure are obviously of partial state feedback control type.

Still, there is a lack in the systematical approach of state controller design and also of the analytical relations for controller and state observer parameters setting.

The purpose of this paper is to present a design methodology for sensorless speed control of two-inertia system with state controller. For design

guidelines validation are presented simulations for different physical parameters values of two-inertia system.

2. Dynamic model of mechanical two-inertia system (TIS)

Figure 1 shows the schematic diagram of the resilient two-inertia system, where we have the following mechanical quantities and parameters:

- m_e, m_t, m_L electromagnetic, torsion and external load torques, [Nm];
- J_M, J_L motor and load inertia, [Kgm²];
- D_M, D_L motor and load viscous damping constants, [Nms/rad];
- k_t torsion stiffness constant [Nm/rad];
- D_t torsion damping constant [Nms/rad].

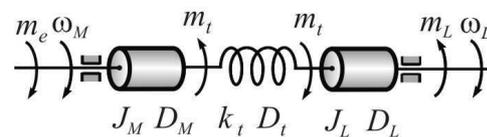


Fig. 1. Schematic diagram of an elastic two-inertia system

The torsion torque has two components proportional with differences between motor and load angles and motor and load angular speed, according to the relation:

$$m_t = k_t (\theta_M - \theta_L) + D_t (\omega_M - \omega_L),$$

The linear model of the two-inertia mechanical system, based on a signal graph diagram, is presented in Figure 2. From here, after eliminating of the auxiliary variable x , which corresponds to a quantity proportional with angles difference, $\Delta\theta$ we obtain the following equations describing dynamic behavior of the controlled plant:

$$\dot{\omega}_M = (D_M / J_M) \omega_M + (m_e - m_t) / J_M, \quad (1)$$

$$\dot{m}_t = k_t (\omega_M - \omega_L) - (D_t J_M / J_M J_L) m_t +$$

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$$+(D_t / J_M)m_e + (D_t / J_L)m_L, \quad (2)$$

$$\dot{\omega}_L = (D_L / J_L)\omega_L + (m_t - m_L) / J_L, \quad (3)$$

where $J_{ML} = J_M + J_L$.

Remark that the two-inertia system can be approximated as a rigid body if the value of parameter k_t , torsion stiffness constant, tends to infinity. In order to simplify the design of the sensorless control two-inertia drive system in the above Equations motor, load and torsion damping constants D_M, D_L, D_t are neglected.

3. Structure of two-inertia sensorless control system

The functional subsystems of the control structure contains (see Figure 3):

– a field oriented controlled permanent magnet synchronous machine (FOC-PMSM);

– a state controller (SC) for regulation of the two-inertia mechanical quantities;

– an outer loop with an I+PI controller for load velocity regulation (LVC);

– a minimum order (Gopinath) state observer (SO).

In order to achieve a fast response with high performances for electromagnetic torque was used a control strategy of the currents in rotor d-q reference frame with forward decoupling.

The PI type current controllers are designed with modulus criterion (Kessler version). Consequently the block FOC-PMSM can be approximately modeled by an equivalent first order system with the transfer function:

$$G_{0i}(s) = \frac{m_e(s)}{i_q^*(s)} = \frac{k_m / k_i}{sT_0 + 1} = \frac{k_0}{sT_0 + 1}, \quad (4)$$

where k_m is torque constant of PMSM, k_i is the gain factor of the current transducers and T_0 is equivalent time constant of the current loops (2 msec in our case)

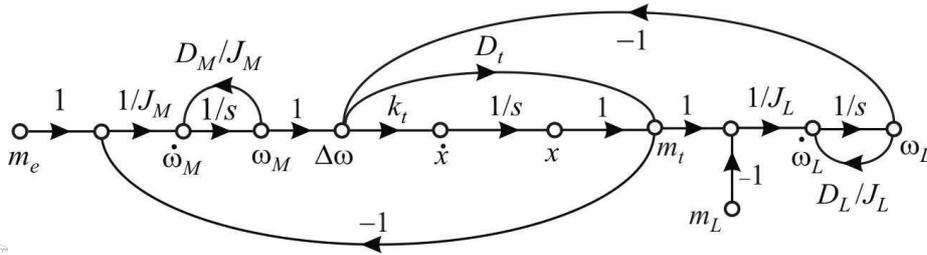


Fig. 2. Signal graph diagram of an elastic two-inertia system

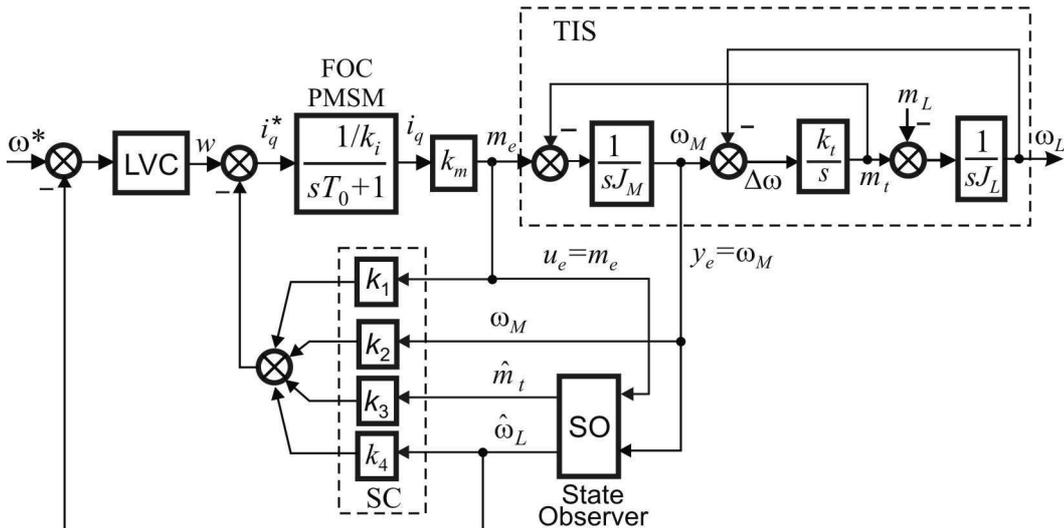


Fig. 3. Block diagram of the sensorless state control system and the linear simplified plant model

By adopting the state vector $x(t)=[m_e \ \omega_M \ m_t \ \omega_L]^T$, and the input $u(t)=i_q^*$, will have the model of the plant in matrix form

$$\dot{x}(t) = \mathbf{A}x(t) + \mathbf{b}u(t), \quad (5)$$

where

$$\mathbf{A} = \begin{bmatrix} -1/T_0 & 0 & 0 & 0 \\ 1/J_M & 0 & -1/J_M & 0 \\ 0 & k_t & 0 & -k_t \\ 0 & 0 & 1/J_L & 0 \end{bmatrix}, \quad (6)$$

$$\mathbf{b} = [k_0/T_0 \ 0 \ 0 \ 0]^T. \quad (7)$$

Note that in this model the motor load, m_L and torsion damping constants, D_t , are neglected.

The P type state controller has the structure shown in Figure 3, wherefrom results that the input of the plant will be now:

$$u(t) = w(t) - \mathbf{k}^T \mathbf{x}(t), \quad (8)$$

where

$$\mathbf{k}^T = [k_1 \ k_2 \ k_3 \ k_4], \quad (9)$$

is transposed vector of the state controller coefficients. Replacing (7) in (6), after applying Laplace transform is obtained the transfer function matrix related to the reference w , described by the expression:

$$\mathbf{G}(s) = \frac{\mathbf{x}(s)}{w(s)} = \begin{bmatrix} s^3 J_M J_L / k_t + s J_{ML} \\ s^2 J_L / k_t \\ s J_L \\ 1 \end{bmatrix} \frac{k_0}{P_0(s)},$$

where

$$\begin{aligned} P_0(s) &= a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0, \\ a_4 &= J_M J_L T_0 / k_t, \ a_3 = J_M J_L (k_0 k_1 + 1) / k_t, \\ a_2 &= T_0 J_{ML} + k_0 k_2 J_L / k_t, \\ a_1 &= (k_0 k_1 + 1) J_{ML} + k_0 k_3 J_L, \\ a_0 &= k_0 (k_2 + k_4). \end{aligned}$$

State control is combined with a classical control structure based on a I+PI type controller (Figure 4), having load velocity as feedback.

The transfer function $G_{w\omega L}(s)$ in Figure 4 has the expression:

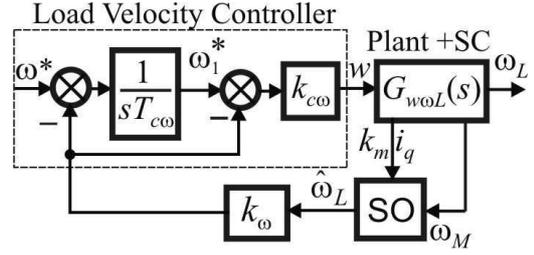


Fig. 4 Block diagram of the Load Speed Control System

$$G_{w\omega L}(s) = \frac{\omega_L(s)}{w(s)} = \frac{k_0}{P_0(s)}. \quad (10)$$

The choice of load speed as a feedback quantity for the controller placed on forward path is a very important design option. Simulation made for the control of two-mass systems with classical motor speed feedback PI controller shown very poor performances even the tendency of stability loss [6].

4. Design of the control system

Pole-placement is achieved by using the extended version of the modulus criterion (EMC) [5], applied to the transfer function of the closed-loop system shown in Figure 4. Because we have six setting parameters ($k_1, k_2, k_3, k_4, k_{c\omega}$ and $T_{c\omega}$) and applying of EMC yields only four equation results that, two parameters can be free chosen. The studies emphasize that the design is simplified if the values of the state controller parameters fulfill the condition $k_2 + k_4 = 0$. The second parameter that can be free chosen is one of the coefficients k_1 or k_3 , which will be set equal to 0. For simplicity we suppose first $k_3 = 0$.

The design procedure has two steps. First are evaluated parameters $k_1, k_2, k_3, k_4, k_{c\omega}$ by applying EMC to the transfer function of internal loop with reference ω_1^* (see Figure 5). In this way for these parameters following relations are obtained [5,6]:

$$k_1 = (\sqrt{2\alpha T_0 / T_t} - 1) / k_0, \quad (11)$$

$$k_2 = (\alpha - 1) \frac{T_0 J_M}{k_0 T_t^2}, \quad k_{c\omega} = \frac{T_0 J_{ML}}{k_0 k_0 T_t^2}, \quad (12a,b)$$

where $T_t = \sqrt{J_M J_L / (k_t J_{ML})}$, is the equivalent torsion time constant and $\alpha = 2 + \sqrt{2} \cong 3.41$. The other state controller parameters are evaluated with relations:

$$k_3 = 0 \text{ and } k_4 = -k_2 = (1 - \alpha) \frac{T_0 J_M}{k_0 T_t^2}. \quad (13a,b)$$

In the second step is obtained the computing relation for controller's parameter, integral time constant of I+PI controller. For this purpose we use the optimization conditions of EMC applied to the transfer function of the closed-loop, with reference ω^* (Figure 4):

$$G_0(s) = \frac{1}{k_\omega} \frac{1}{P_1(s)}, \quad (14)$$

where

$$P_1(s) = 1 + c_1 s + c_2 s^2 + c_3 s^3 + c_4 s^4 + c_5 s^5$$

$$c_1 = T_{c\omega}, \text{ and } c_i = b_i T_{c\omega}, \text{ for } i = 2, 5.$$

Now we have a single unknown parameter, $T_{c\omega}$ and four optimization EMC conditions. Solving first equation for this parameter we obtain

$$T_{c\omega} = 2\sqrt{2\alpha} T_t. \quad (15)$$

Relation (10) for the computing torque feedback coefficient restricts the application range of the design procedure. To avoid positive feedback from electromagnetic torque must be fulfilled condition $k_1 > 0$, or equivalent

$$\sqrt{2\alpha} T_0 = 2.61 T_0 > T_t. \quad (16)$$

If this inequality is not fulfilled, is necessary to introduce feedback from torsion torque, which implies $k_3 \neq 0$. Studies show in this case that we can a priori adopt, $k_1 = 0$.

Solving in the same way as previous for transfer function (10) we obtain the following relations for computing of setting parameters in case $k_1 = 0$;

$$k_2 = \frac{T_0 J_{ML} k_t}{J_L} \left(\frac{T_t^2}{2T_0^2} - 1 \right) / k_0, \quad k_4 = -k_2, \quad (17)$$

$$T_{c\omega} = 4\alpha T_0, \quad k_3 = \frac{J_{ML}}{J_L} \left(\frac{T_t^2}{2T_0^2} - 1 \right) / k_0, \quad (18a,b)$$

$$k_{c\omega} = \frac{T_t^2 J_{ML}}{4\alpha^2 k_\omega k_0 T_0^3}. \quad (19)$$

5. State observer design

The control structure with state controller needs real time acquisition for the state quantities of elastic mechanical transmission. Due to the increasing of the

automation charges when the transducers are used, in case of some state variables, as a consequence the mechanical sensors shall not be used. In addition, some of the mechanical quantities (e.g. the torsion torque), can not be accessed by some direct measurements using the transducers.

The state observer represents a modern, real time acquisition method of the mechanical quantities, which are needed in the controlling scheme.

In order to collect data without a mechanical sensor, a different observer (e.g. Gopinath or an extended Kalman filter) shall be used.

In this paragraph, the Gopinath observer structure and design, which is used to estimate the states of the torsion load electromechanical subsystem, is to be presented.

The fixed part of the model can be calculated starting from the torsion load subsystem model, by extending and reordering of the state vector, which shall have the following expression:

$$\mathbf{x} = [m_t \quad \omega_L \quad m_L \quad \omega_M]^T = [\mathbf{x}_e^T \quad y]^T, \quad (20)$$

where $\mathbf{x}_e = [m_t \quad \omega_L \quad m_L]^T$ is the vector of states which needs to be estimated, and $y = \omega_M$ is the system output (a measurable quantity).

Adopting the vector of the input variable $u = m_e$, (m_e is the electromagnetic torque of the SM), the model can be written in the compact form:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}u \\ y &= \mathbf{C}\mathbf{x} \end{aligned} \quad (21 \text{ a,b})$$

Hereto, the output of a fixed part, which represents the second input of the observer, is the angular speed (ω_M).

The matrices of the estimated system are:

$$\mathbf{A} = \begin{bmatrix} 0 & -k_t & 0 & k_t \\ 1/J_L & 0 & -1/J_L & 0 \\ 0 & 0 & 0 & 0 \\ -1/J_M & 0 & 0 & 0 \end{bmatrix}, \quad (22)$$

$$\mathbf{B} = [0 \quad 0 \quad 0 \quad 1/J_M]^T, \quad \mathbf{C} = [0 \quad 0 \quad 0 \quad 1]. \quad (23a,b)$$

Note that the state space models used for estimation and, respectively for state controller are different. In order to develop the state observer structure, first we adopt auxiliary matrices:

$$\mathbf{A}_{11} = \begin{bmatrix} 0 & -k_t & 0 \\ 1/J_L & 0 & -1/J_L \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{A}_{12} = \begin{bmatrix} k_t \\ 0 \\ 0 \end{bmatrix}, \quad (24a, b)$$

$$A_{21} = \begin{bmatrix} -1/J_L & 0 & 0 \end{bmatrix}, A_{22} = 0, \quad (25a, b)$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T, B_2 = 1/J_M, \quad (26a, b)$$

which are obtained by splitting of matrices A and B of the estimated model, in the way imposed by the splitting of state variables vector x . The general equations of the Gopinath observer are [5, 13]:

$$\dot{z}(t) = F z(t) + G u(t) + H y(t), \quad t \in \mathbf{R}^+ \quad (27)$$

$$x_e(t) = z(t) + L y(t), \quad (28)$$

where z is the state variables vector of the Gopinath observer and

$$F = A_{11} - L \cdot A_{21}, G = B_1 - L \cdot B_2, \quad (29a, b)$$

$$H = A_{12} - L \cdot A_{22} + F \cdot L, L = \begin{bmatrix} l_1 & l_2 & l_3 \end{bmatrix}^T, \quad (30a, b)$$

wherefrom we obtain

$$F = \begin{bmatrix} l_1/J_M & -k_t & 0 \\ [(1/J_L) + (l_2/J_M)] & 0 & -1/J_S \\ l_3/J_M & 0 & 0 \end{bmatrix}, \quad (31)$$

$$G = \begin{bmatrix} -l_1/J_M & -l_2/J_M & -l_3/J_M \end{bmatrix}^T, \quad (32)$$

$$H = \begin{bmatrix} k_t(1-l_2) + l_1^2/J_M \\ l_1[(1/J_L) + l_2/J_M] - l_3/J_L \\ l_1 l_3/J_M \end{bmatrix}. \quad (33)$$

The dynamic behavior of the state observer (27) is established by the matrix F , which is characterized by the characteristic polynomial:

$$P_0(s) = \det(sI - F)$$

$$P_0(s) = s^3 - s^2 \frac{l_1}{J_M} + s k_t \left(\frac{1}{J_L} + \frac{l_2}{J_M} \right) - c_e \frac{l_3}{J_M J_L}.$$

In order to get most proper values for the F matrix, an optimizing polynomial is introduced:

$$P_{opt}(s) = s^3 + 2 \omega_0 s^2 + 2 \omega_0^2 s + \omega_0^3,$$

which might be obtained through the extended modulus criterion (EMC) application, in terms of a free selection for one parameter (ω_0 as in our case); moreover, the natural pulsation (ω_0) can be taken based on the designer experience, the final result to lead to the fastest answer of the observer.

By the polynomials' P_0 and P_{opt} alike powers coefficients identification, the new designing relation should be deduced in order to get the L matrix parameters:

$$l_1 = -2 \omega_0 J_M, \quad (34)$$

$$l_2 = \left(2 \omega_0^2 / k_t - 1 / J_L \right) J_M, \quad (35)$$

$$l_3 = -(J_M J_S / k_t) \omega_0^3. \quad (36)$$

Replacing the relations (31-33) in equation (27), the final form of the Gopinath observer equations for the measurements of elastics mechanical transmissions can be obtained:

$$\begin{aligned} \dot{z}_1 &= (l_1/J_M)z_1 - k_t z_2 - (l_1/J_M)m_e + \\ & [k_t(1-l_2) + (l_1^2/J_M)]\omega_M \end{aligned} \quad (37)$$

$$\begin{aligned} \dot{z}_2 &= [(1/J_L) + (l_2/J_M)]z_1 + (1/J_L)z_3 - \\ & (l_2/J_M)m_e + l_1[(1/J_M) + (l_2/J_L)]\omega_M - (l_3/J_L)\omega_M \end{aligned} \quad (38)$$

$$\dot{z}_3 = (l_3/J_M)z_1 - (l_3/J_M)m_e + (l_1 l_3/J_M)\omega_M \quad (39)$$

$$\hat{m}_t = z_1 + l_1 \omega_M, \quad (40)$$

$$\hat{\omega}_L = z_2 + l_2 \omega_M, \quad (41)$$

$$\hat{m}_L = z_3 + l_3 \omega_M. \quad (42)$$

The introduced observer can be simplified, if the estimated measurements (40-42) are introduced in equations (37-39). Thus, results:

$$\dot{z}_1 = [\omega_M - \hat{\omega}_L - l_1(m_e - \hat{m}_t)/(c_e J_M)]c_e, \quad (43)$$

$$\dot{z}_2 = [\hat{m}_t - \hat{m}_L - l_2 J_S(m_e - \hat{m}_t)/J_M]/J_S, \quad (44)$$

$$\dot{z}_3 = -l_3(m_e - \hat{m}_t)/J_M, \quad (45)$$

which, together with the equations (40-42), represents the final form of the Gopinath observer.

6. Analysis of the sensorless control system

In order to analyze the dynamic behavior of the sensorless control system with state observer simulations a lot of simulations were realized in MATLAB–SIMULINK.

The rated data of the synchronous motor employed in simulation are:

$$p=2, R_s = 0.98 \Omega, \Psi_e = 0.174 \text{ Wb},$$

$$L_d = L_q = 0.015 \text{ H}, J_M = 0.0087 \text{ Kgm}^2.$$

The parameters of the current field oriented control structure approximated as a first order element have the values $k_0 = 1.044$ and $T_0 = 0.002 \text{ sec}$.

To analyze various cases, in simulation, the parameters of the two-mass system, k_t and J_L , are adopted variable in a large range of values.

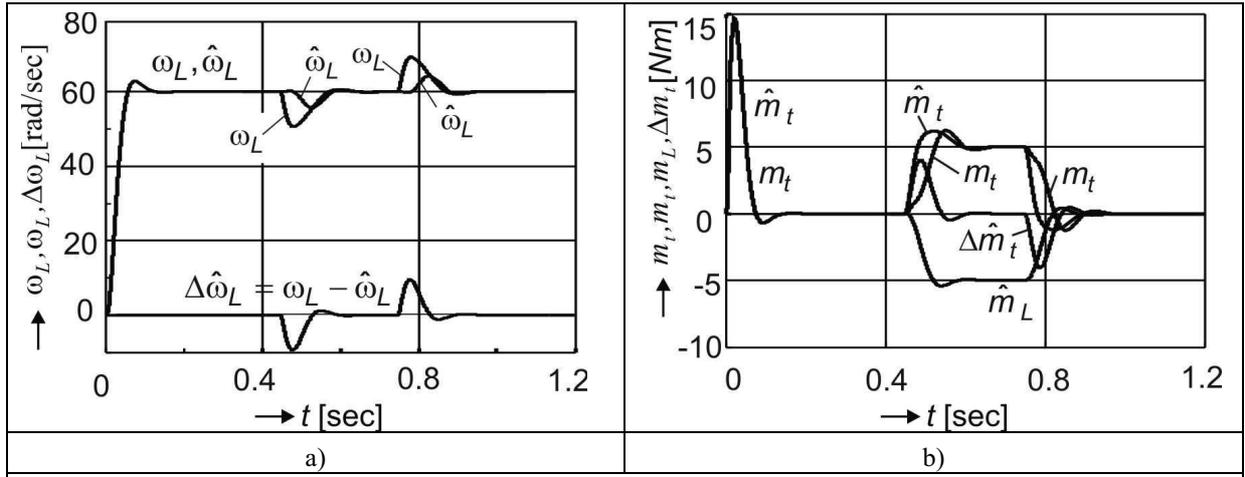


Fig. 5 Responses to step reference and torque load disturbance for a two inertia electrical drive with $k_t = 50$ Nm/rad and $J_L = J_M$, a) the load speed (real ω_L , estimated $\hat{\omega}_L$ and estimation error $\Delta\hat{\omega}_L$), b) torsion torque (real m_t , estimated \hat{m}_t and estimation error $\Delta\hat{m}_t$) and estimated load torque \hat{m}_L

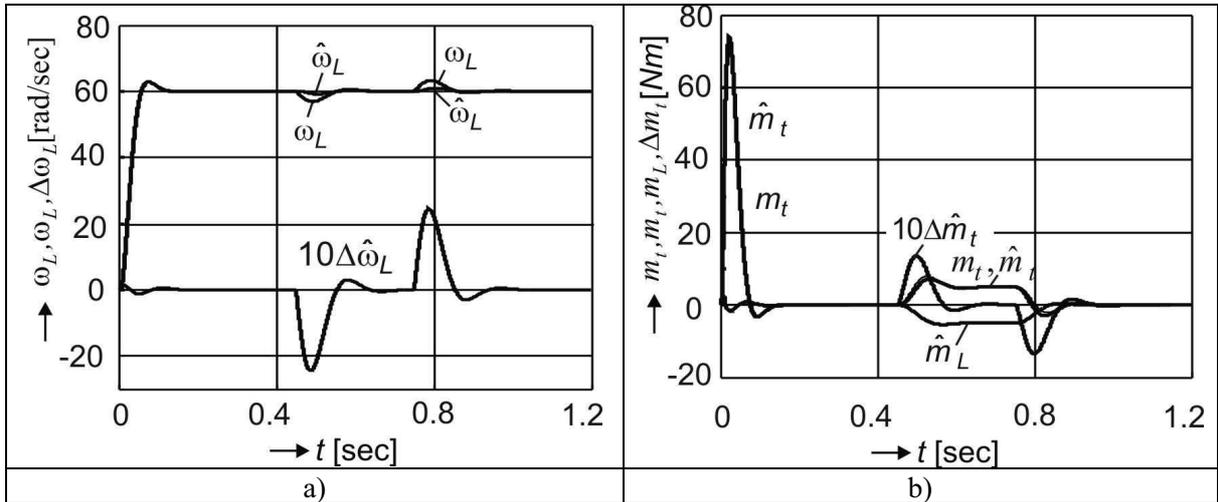


Fig. 6 Responses to step reference and torque load disturbance for a two inertia electrical drive with $k_t = 50$ Nm/rad and $J_L = 5J_M$, a) the load speed (real ω_L , estimated $\hat{\omega}_L$ and estimation error $\Delta\hat{\omega}_L$), b) torsion torque (real m_t , estimated \hat{m}_t and estimation error $\Delta\hat{m}_t$) and estimated load torque \hat{m}_L .

In simulations we study the dynamic behavior of the combined control structure with load speed feedback PI and state controllers. It is assumed that the estimator is on-line connected to the control system, that is the feedback quantity will be the estimated ones instead of the real.

As we can see in Figures 5, 6, 7, 8, the estimation quality is good. The estimation errors are greater for load disturbance and if load and motor inertia is strongly different or the torsion stiffness constant has low values.

7. Conclusions

The paper presents sensorless state-space control of two-inertia drive system with resilient coupling. The control structure contains an I+PI controller for load speed regulation and a state feedback controller for effective vibration suppression of the elastic coupling. Mechanical state variable of two-inertia drive are obtained by using a linear minimum-order (Gopinath) state observer.

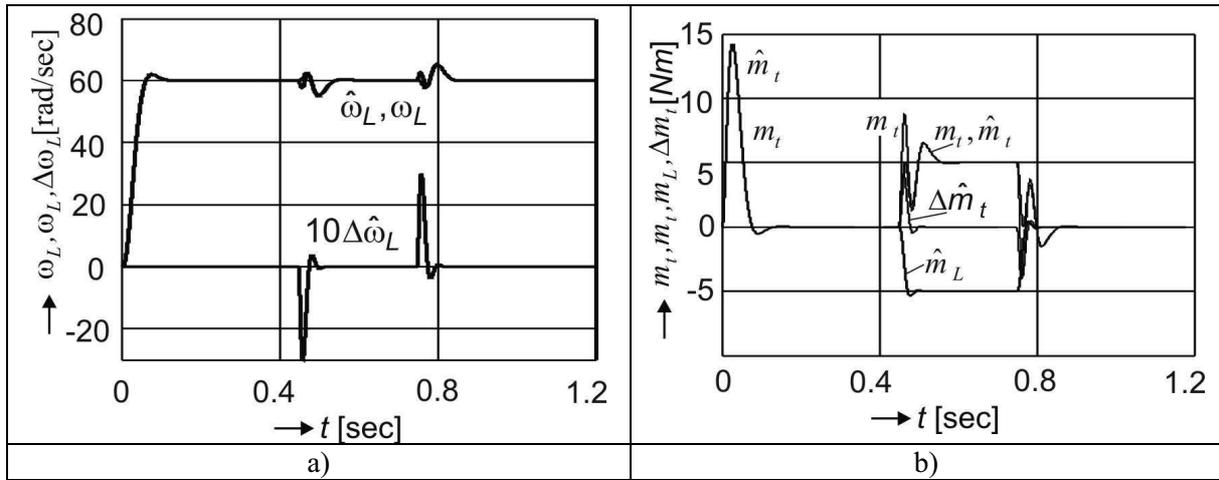


Fig.7 Responses to step reference and torque load disturbance for a two inertia electrical drive with $k_t = 500$ Nm/rad and $J_L = J_M$, a) the load speed (real ω_L , estimated $\hat{\omega}_L$ and estimation error $\Delta\hat{\omega}_L$), b) torsion torque (real m_t , estimated \hat{m}_t and estimation error $\Delta\hat{m}_t$) and estimated load torque \hat{m}_L .

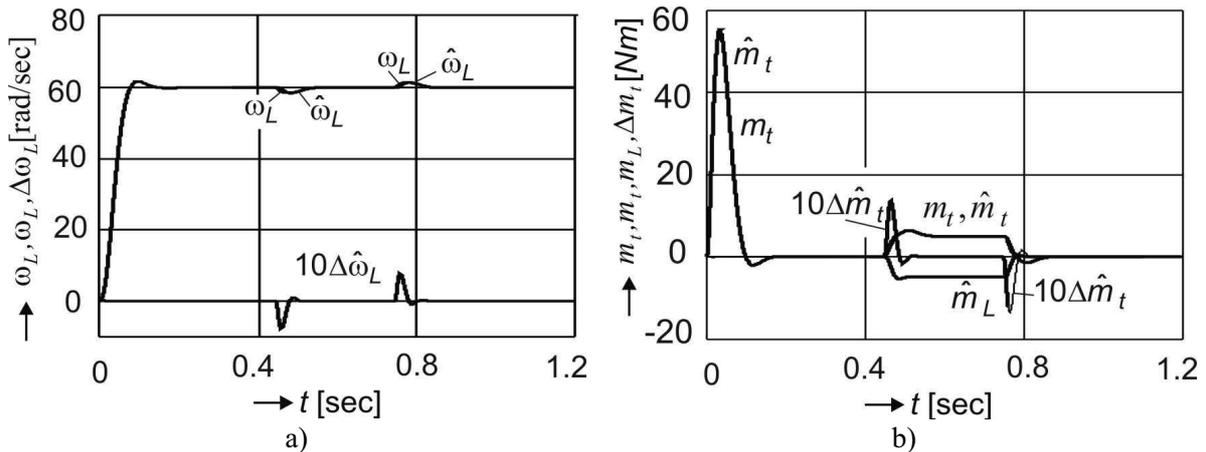


Fig.8 Responses to step reference and torque load disturbance for a two inertia electrical drive with $k_t = 500$ Nm/rad and $J_L = 5J_M$, a) the load speed (real ω_L , estimated $\hat{\omega}_L$ and estimation error $\Delta\hat{\omega}_L$), b) torsion torque (real m_t , estimated \hat{m}_t and estimation error $\Delta\hat{m}_t$) and estimated load torque \hat{m}_L .

The design of the combined (I+PI and state feedback) controller is achieved with the extended version of the modulus criterion [5]. The dynamic behavior of presented control structure has been examined, for different conditions, using MATLAB/SIMULINK simulation.

The simulations made for proposed sensorless control structure designed with EMC, show a very good dynamic properties.

8. References

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