

## OPTIMIZATION STRATEGIES TO INCREASE ELECTRICAL DISTRIBUTION NETWORKS ROBUSTNESS 20 KV AND 110 KV FROM COUNTY MUREȘ

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### ABSTRACT

*The paper aims to present a mathematical model to optimize power distribution network graph, in terms of increasing its robustness, ie to reduce the risk of destruction (its removal from service – accidentally or intentionally), with applications to the distribution networks 20 kV and 110 kV, County Mures.*

**Keywords:** risk, scales free networks, optimization, mathematical model, application

### 1. Introduction

The graf of a network with ”n” nodes is given by the parameters:

$$\begin{matrix} K & | & k_1 & k_2 & \dots & k_i \\ NK & | & nk_1 & nk_2 & \dots & nk_i \end{matrix} \quad (1)$$

where, K and NK vector components are:

$nk_i$  |  $i = 1, n$  – is the number of nodes with  $k_i$  links (power lines);

$k_i$  |  $i = 1, n$  – links (power lines).

An indicator of the robustness of this graph is critical fraction ” $f_c$ ” which is calculated with the formula:

$$f_c = 1 - \frac{1}{\gamma - 1} \quad (2)$$

where:

$$\gamma = \Delta(k^2) / \Delta(k) \quad (3)$$

and

$$\Delta(k^2) = \sum_i^n k_i^2 * nk_i / \sum_i^n nk_i \quad (4)$$

$$\Delta(k) = \sum_i^n k_i * nk_i / \sum_i^n nk_i \quad (5)$$

$f_c = (0.56-0.80)$  means that such a network retains its essential quality – interconnectivity - until a random fraction of nodes destroyed does not exceed about 70%. This property is due to well defined in a number of hubs (dependent  $\gamma$ ).

The robustness of a network can grow by:

$$\text{Max} \{f_c\} \quad \text{with} \quad 0 \leq f_c \leq 1 \quad (6)$$

Respectively by setting the optimal number of nodes  $NK_i^{opt}$  |  $i = 1, n$  for  $K_i$  |  $i = 1, n$  constant.

Based on the graphic representation in coordinates logarithmic the parameters of a given graph by  $K_i$  and  $NK_i$ , can be graphically represented by Figure 1, where:

„A” – represents the graph of function:

$$\log(K) = F[\log(NK)] \quad (7)$$

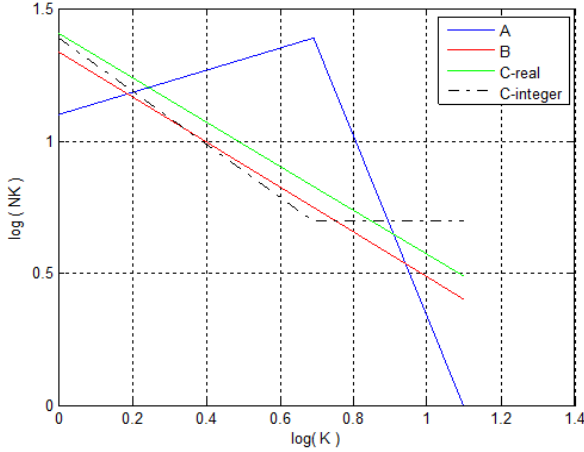
„B” – represents the graph function that best approximates the right-broken ”A”.

It can be proved mathematically [2] that increases network robustness, for values of  $NK_i^{opt}$  |  $i = 1, n$  in which the logarithmic representation will get it right „C” as, close to right „B”, or:

„C” – represents the graph of function:

$$\log(K) = F[\log(NK^{optim})] \quad (8)$$

For the recommended values of  $\gamma = (2-3)$ ,



**Fig. 1. Parameters variation of robustness of example:  $k = [1 \ 2 \ 3]$  and  $NK = [3 \ 4 \ 1]$**

Thus, to achieve a fixed  $f_c$  is equivalent to search, we'll proceed in two steps:

a) We'll found, in logarithmic coordinates, a line having as minimal slope  $S$ , so that  $tgS = -\gamma$ , assuring a target critical fraction  $f_c$  ;

b) We'll have to obtain the target graph  $G_F$ , from the initial  $G_o$  graph of the studied power grid. In other words, we'll have to identify the elementary transformations from  $G_o$  to  $G_F$ . We decide to choose for adjustments, only the nodes with  $N_k^0 < N_k^F$ , considering that in the contrary case, the additional connections' number can be eliminated.

## 2. The mathematical optimization model

Based on the theoretical aspects, the mathematical model consists in determining the optimal values for  $NK_i^{opt} \mid i=1,n$  which for  $K_i$  constant, will approximate the right „C” with a maximum according to the right „B” – which approximates the best linear broken line „A”.

### 2.1. Variables by optimized

Shaping this three straight mathematical, their equations are:

The broken line „A” representing the given graph, is:

$$\begin{aligned} a &= \log(K_i) \\ b &= \log(Nk_i) \end{aligned} \quad (9)$$

The straight line of approximation of the „A” will be the „B” equation:

$$Z_B = m * a + n \quad (10)$$

where:

$$m = coef(1)$$

$$n = coef(2)$$

coefficients are given by polyfit function of Matlab, or:

$$coef = polyfit(a,b,1) \quad (11)$$

The equation of straight line „C” which has the best coincidence with straight line „B” will be as follows:

$$Z_C = m * x1 * a + n * x2 \quad (12)$$

where  $x1$  and  $x2$  variables represent the components by optimized of variables vector  $X$ .

$$X = [x1 \ x2] \quad (13)$$

The components of  $Z_C = F(x1 \ x2)$ , are actually components of the new optimized vector:

$$NK^{opt} \mid nk_1^{opt} \ nk_2^{opt} \ \dots \ nk_n^{opt} \quad (14)$$

with

$$nk_i^{opt}(X) \mid i=1,n$$

### 2.2. Objective function

The objective function is given by maximizing the critical fraction „ $f_c$ ”:

$$Max \{f_c\} \quad \text{with} \quad 0 \leq f_c \leq 1 \quad (15)$$

where

$$f_c = 1 - \frac{1}{\gamma - 1} \quad (16)$$

or

$$Min \{1/\gamma - 1\} \quad (17)$$

where:

$$\gamma = \Delta^{opt}(k^2) / \Delta^{opt}(k) \quad (18)$$

and

$$\Delta^{opt}(k^2) = \frac{\sum_i^n k_i^2 * nk_i^{opt}}{\sum_i^n nk_i^{opt}} \quad (19)$$

$$\Delta^{opt}(k) = \frac{\sum_i^n k_i^{opt} * nk_i}{\sum_i^n nk_i^{opt}} \quad (20)$$

### 2.3. Constraints of the model

The constraints model can have two types:

- linear constraints by form:

$$x1 \geq 0 \quad x2 \geq 0 \quad (21)$$

- nonlinear constraints imposed on the number of nodes in the graph optimized to remain the same as the original graph, or:

$$\sum Nki^{optim} = \sum Nki^{given} \quad (22)$$

Also, other ways can be introduced in the optimization procedure:

- Verify the  $\sum ki \cdot Nki^{optim}$  product parity. If the product isn't a twin number, we must adjust the  $Nki^{optim}$  values. In aim to not modify essentially the line slope, these changes will be made on the  $Nki^{optim}$  corresponding to the small values of  $ki$ ;
- If the optimized graph doesn't assure the target  $f_c$ , a strategy to growth the line's slope is to renounce to the weakest connected nodes (e.g., the nodes having one connection).

### 3. Solving the model

The problem is formulated as a nonlinear optimization problem solved with the specialized software package MATLAB 7.0, *fmincon* procedure.

Follows the optimal values of the variables  $x1$  and  $x2$  for which coefficients the straight line „C” represents the optimal number of nodes  $Nki^{opt} | i=1,n$  with  $ki$  connections for critical deviations „ $f_c$ ” is minimal.

In the iterative optimization process has been a real component conversion always of straight line „C-real” in the vector components optimized  $NK^{optim}$  whole, represented by the broken line „C-integer”

### 4. Applications. Results and discussion.

For 20 kV network of Mures County, graph network has the following initial parameters:

$$\begin{aligned} K &= [1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7]; \\ NK^{given} &= [20 \ 15 \ 3 \ 0 \ 2 \ 1 \ 1]; \end{aligned} \quad (23)$$

with the initial value  $f_c = 0.4876$ .

The parameters resulting from optimization are:

$$\begin{aligned} K &= [1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7]; \\ NK^{optim} &= [24 \ 8 \ 4 \ 2 \ 2 \ 1 \ 1]; \end{aligned} \quad (24)$$

According to the parity condition, the final, corrected result is:

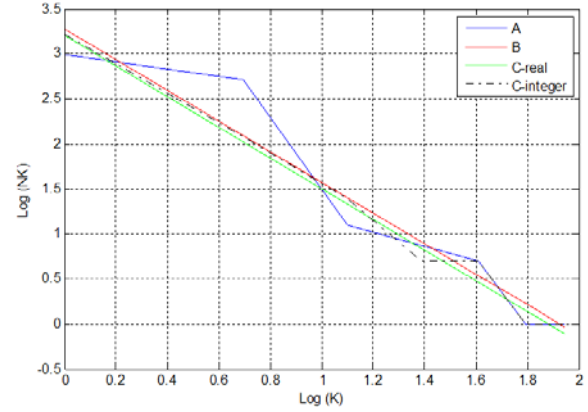
$$\begin{aligned} K &= [1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7]; \\ NK^{optim} &= [23 \ 9 \ 4 \ 2 \ 2 \ 1 \ 1]; \end{aligned} \quad (25)$$

with the final value  $f_c = 0.5284$ .

The optimized values are:

$$\begin{aligned} xopt_1 &= 1.0045 \\ xopt_2 &= 0.9808 \end{aligned} \quad (26)$$

Graphs of the straight lines „A”, „B” and „C” are shown in Figure 2.



**Fig. 2. Parameters variation in robustness optimization process for 20 kV of Mures County**

For 110 kV network of Mures County, graph network has the following initial values:

$$\begin{aligned} K &= [1 \ 2 \ 3 \ 4 \ 5 \ 6]; \\ NK^{given} &= [4 \ 19 \ 5 \ 0 \ 3 \ 1]; \end{aligned} \quad (27)$$

with the initial value  $f_c = 0.5063$ .

The parameters resulting from optimization are:

$$\begin{aligned} K &= [1 \ 2 \ 3 \ 4 \ 5 \ 6]; \\ NK^{optim} &= [11 \ 6 \ 5 \ 4 \ 3 \ 3]; \end{aligned} \quad (28)$$

According to the parity condition, the final, corrected result is:

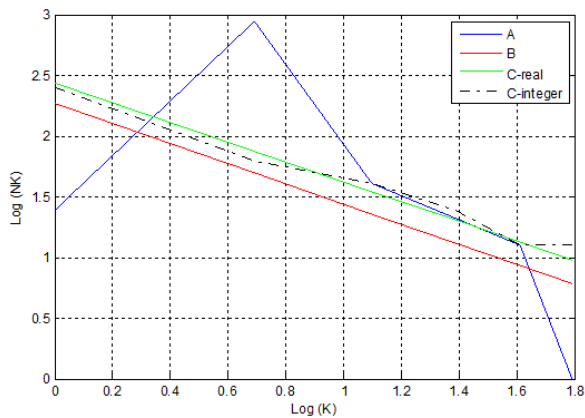
$$\begin{aligned} K &= [1 \ 2 \ 3 \ 4 \ 5 \ 6]; \\ NK^{optim} &= [10 \ 7 \ 5 \ 4 \ 3 \ 3]; \end{aligned} \quad (29)$$

with the final value  $f_c = 0.6375$ .

The optimized values are:

$$\begin{aligned} xopt_1 &= 0.9798 \\ xopt_2 &= 1.0729 \end{aligned} \quad (30)$$

Graphs of the straight lines „A”, „B” and „C” are shown in Figure 3.



**Fig. 3. Parameters variation in robustness optimization process for 110 kV of Mures County**

### 5. Conclusions

The ideas presented in our paper offers useful tools to extend step by step one vulnerable network to the nearest robust network. Measuring the critical fraction's variance is hence an appropriate pointer helping to choose optimal enlargement strategies, consisting in taking decisions on the elementary enlargements' order. Analyzing the applications' result, we'll must focus adjusting the mathematical results with the real enlargement possibilities: the realistic level of the grids' robustness can be achieves also by renouncing to make all, mathematically necessary enlargements. Also, these tools give the departing point for the economically acceptable solutions, depending to the real geographic locations o the power grid's nodes.

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