RESEARCH ON ESTABLISHING THE RANK AND QUOTIENT OF FUNCTIONS IN PRODUCT VALUE ANALYSIS/ENGINEERING

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ABSTRACT

The constructive conception of a product results from uniting subsystems with basic usage values. These basic usage values make up the functions of the product. The notion of product function is the basic notion that product value analysis/value engineering(VA/VE) operates with, and function analysis together with creative thinking constitutes „the oxygen of value engineering". The present paper defines the notion of rank of a product function, establishes the formula for calculating its value and it reviews some ways of determining the levels of importance of product functions, with the aim of proposing a new distribution of the importance of these functions within the total usage value. Establishing the rank of a function can be reduced to the issue of comparing product functions by experts, consumers, team members for VA/VE. Subsequently, the ensuing results are subjected to adequate mathematical operations in order to determin the levels of importance and the quotients of each function within the product ussage value, as well as the distribution of these quotients. Due to the fact that the quota or quotient of a function within the product usage value plays an important role in conceiving and designing products, more precisely, in the economical shaping of functions, the distribution law to which this parametre is subjected is also very important. A critical study of the methods currently used to determine function quotients shows that these methods conduct to a linear distribution of these quotients, and, under these circumstances, the ratio between the highest level of importance and the lowest level of importance is equal to the number of functions – number that is very high indeed for complex products. On the other hand, it is rightly assumed that there is a considerable number of products for which the functions do not follow a linear distribution. The Zipf distribution or its generalised form, the Pareto-Zipf-Mandelbrot distribution, can be an alternative to the linear distribution. This distribution is valid in very many fields, of which most relevant for the present paper is the field of prices.

Key words: product function, function rank, function quotients, quotient distribution, Pareto-Zipf-Mandelbrot distribution

1. Introduction

The constructive conception of a product results from uniting subsystems with basic usage values. The basic usage values are the results of a division in depth of partial product usage value until it reaches a point where division is no longer possible. These basic usage values are the product functions. The notion of product function is the fundamental notion value analysis/value engineering (VA/VE) operates with, and function analysis together with creative thinking constitutes "the oxygen of value engineering"[1].

Evaluating product functions is necessary in VA/VE for two apparently contradictory reasons:

1. on the one hand, abstract generalisation is necessary, distancing oneself from the concrete achievement of the researched objective, the most edifying example in this sense being the achievement of mechanic flight: "from the earliest days man has tried to fly by imitating birds. Only by forgetting the flight of birds and focusing on the function it fulfils – sustainability was man able to achieve the first flight"[1].
2. on the other hand, each function of the object must be analysed in relation to its concrete way of achievement.

2. Function rank and quotient in product value analysis/value engineering

The present paper defines the notion of product function rank and reviews a few ways of determining it, with an aim to propose a new distribution of these functions quotients within the total product usage value.

The function rank is defined as being the number that indicates its position within the decreasingly disposed array of function quotients in the product usage value. Although at first glance it would seem that one must know the product function quotient in order to establish the function rank, things are exactly the opposite because these quotients are not known, whereas establishing a function's rank can be reduced to a matter of comparing product functions by experts, consumers, and team members for VA/VE. Subsequently, the results are processed by means of appropriate mathematic methods in order to establish the levels of importance of each function, the quotient distribution and the function quotient in the product usage value.

Establishing the importance levels of functions and their quotients in the product usage value . Although this is a relatively simple operation, it plays a very important role in the success of a VA/VE research and it requires a deep knowledge of the product, of the circumstances in which the product works and of the social needs it fulfils. As elementary usage value the function is measured through its technical dimensions. The function usage values are unequal, as each participates differently to the formation of the total usage value. Due to this reason, these values should be coordinated according to this participation[2].

3. Determining function quotients within the product usage value

Determining the function quotients within the product usage value requires comparing different usage values, which are technically measurable by means of different measure units[3]. The studies on VA/VE describe several methods used for determining the importance levels and the function quotients in the product usage. We will present two of these methods.

The first method[2] requires the creation of a matrix(table 1) in which the main and necessary functions are located on the first line(F_j) and on the first column (F_k).

Table 1. Comparing functions using digits 0 and 1

	F1	F2	F3	F4	F5
F1	1	1	0	1	0
F2	0	1	0	0	0
F3	1	1	1	1	1
F4	0	1	0	1	0
F5	1	1	0	1	1
n_j	3	5	1	4	2
q_j	0.20	0.33	0.07	0.27	0.13
r_j	3	1	5	2	4

Function F_j is compared with F_k and if, for example F_j is more important in the product usage value than the function F_k ($F_j > F_k$) in cell kj, digit 1 will be written ($a_{kj} = 1$), and for the opposite case ($F_j < F_k$) in the cell *kj* it will be written $a_{kj} = 0$.

If the functions to be compared cannot be differentiated, additional research is required in order to achieve their differentiation. The main diagonal upper left – lower right (k = j) is complementary to $a_{kj} = 1$. The importance level of a function F_j is given by (1)[2] .

$$n_j = \sum_{k=1}^{N} a_{kj} \qquad (1)$$

If the function comparison and the filling of the table is done correctly then n_j takes on all the values, integers, between 1 and the number N of analyzed functions.

The quotient or the percentage of function F_j in the product usage value is the ratio between the importance level of the function and the sum of importance levels of all the functions (2)[2] .

$$q_j = \frac{n_j}{\sum_{j=1}^{N} n_j} \qquad (2)$$

The function rank, as defined above, is given by (3)

$$r_j = N + d - n_j \qquad (3)$$

in which *d*(from diagonal) is the digit which should be inserted in the cells on the evaluation matrix diagonal.

The assessment performed by individual experts, even if an appropriate mathematic processing, still allows for subjectivity. This is why a collective expertise is necessary, especially in those situations when it is necessary to obtain stable values for different indicators like the typical function quotient [3]. Since each expert will be required to fill in a table identical to table 1, the importance levels of the functions will be given by the average of the importance levels taken from the tables submitted by all the experts (4)[2]

$$q_j = \frac{\sum_{e=1}^{E} n_{je}}{E} = \frac{\sum_e \sum_k a_{kje}}{E} \qquad (4)$$

in which E is the number of experts.

The second method is similar. The difference lies in that when comparing the function pairs, in the cell where each line intersects with the columns it is inserted the index of the function considered to be more important. In the column which indicates the importance level n_j the total number of occurrences of the j index of the F_j function on the j line. An example for the usage of this method can be found in table 2[3].

Table 2. Comparing functions using their indicators

Function		F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	n_j	r_j
Considers quantity	F1	0	1	3	4	5	6	7	1	1	1	1	1	1	1	8	6
Determines consumers	F2	1	0	3	4	5	6	7	2	2	2	2	2	2	2	7	7
Tracks stocks	F3	3	3	0	4	3	3	7	3	3	3	3	3	3	3	11	3
Signals necessity to place an order	F4	4	4	4	0	4	4	4	4	4	4	4	4	4	4	13	1
Communicates monthly flow	F5	5	5	3	4	0	6	7	5	5	5	5	5	5	5	9	5
Communicates final state	F6	6	6	3	4	6	0	7	6	6	6	6	6	6	6	10	4
Tracks meeting orders	F7	7	7	7	4	7	7	0	7	7	7	7	7	7	7	12	2
Compares stocks	F8	1	2	3	4	5	6	7	0	8	8	8	8	8	8	6	8
Determines VAT	F9	1	2	3	4	5	6	7	8	0	9	11	12	13	14	1	13
Makes calculations	F10	1	2	3	4	5	6	7	8	9	0	11	12	13	14	0	14
Marks materials	F11	1	2	3	4	5	6	7	8	11	11	0	12	13	14	2	12
Ensures automatisation	F12	1	2	3	4	5	6	7	8	12	12	12	0	13	14	3	11
Gives discounts	F13	1	2	3	4	5	6	7	8	13	13	13	13	0	14	4	10
Opens documents	F14	1	2	3	4	5	6	7	8	14	14	14	14	14	0	5	9

Out of a number of ten papers on VA/VE consulted, the only one that includes the notion of function rank is table 2 at [3], which denotes that this notion is not common in research on this subject. The reason for introducing the notion of rank will be seised upon in chapter 5.

4. The quotient distribution of product functions obtained by current methods

In [2] it is described a critic study regarding the current methods for determining the importance levels and the function quotients. According to this study:

1. The importance level n_j of a function and its quotient q_j in the usage value displays a linear variation , with a steep incline (lines a and b in figure 1)
2. The α ratio (5)[2] between the highest importance level ($n_{j\ max}$) and the lowest importance level ($n_{j\ min}$)

$$\alpha = \frac{n_{j\max}}{n_{j\min}} \qquad (5)$$

in the case of linear variation is equal to the number of functions N – which is very high for the complex products.

3. Due to the fact that the quotient q_j of the function Fj in the product usage value plays an important role in the product design and creation, specifically in the economic dimension of the functions, the distribution law to which this parameter is subjected is equally important. The VA/VE methodology requires that q_j should be approximately equal to p_j(the cost quotient in the total cost of the product CP). This means that the spend C_j, corresponding to a function F_j, are equal to the product between q_j and CP relation 6[2].

$$C_j = q_j \bullet CP \qquad (6)$$

Therefore, the higher the number of functions N, the most important function could be N times more expensive than the least important function because it would have $n_{jmax} = N = \alpha$, in this case n_j can take all the values, integers between 1 and the number of function N. Table 3 displays the values of the α ratio between the highest and lowest importance levels for different hypothetical products defined by the number of functions (N = 5; 10; 20; 100) and for various sets of digits assigned to a_{kj}.

Table 3. α ratio values

	N=5	N=10	N=20	N=100
a_{kj} = 0 şi 1	5	10	20	100
a_{kj} = 0; 1 şi 2	9	19	39	199
a_{kj} = 1 şi 3	2,6	2,8	2,9	2,98
a_{kj} = 1 şi 5	4,2	4,6	4,8	4,96
a_{kj} = 1; 3 şi 6	5	5,5	5,75	5,96

It can be observed that the elimination of the digit 0 from the set of digits a_{kj} led to a "settling down" of α. N = 100 of functions α is equal to 199, if a_{kj} = 0; 1; 2 and it is 5,96 if a_{kj} = 1; 3; 6.

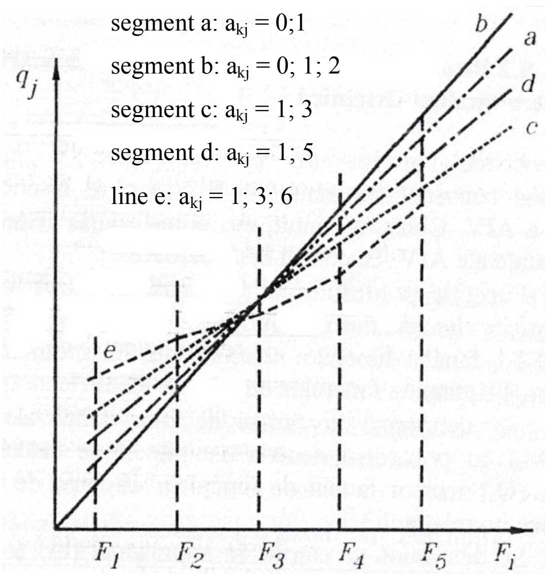

Fig. 1 – Linear distribution of function quotients

Taking into account 5 functions of a product and using the same sets of digits a_{kj} for comparison the graphical representation of the quotient distribution q_j of the functions is displayed in figure 1, a linear distribution [2].

However, as it can be seen in the following sections, it can be assumed that there are products for which the function quotients value do not have a linear distribution.

5. Arguments for using the Pareto-Zipf-Mandelbrot distribution in establishing product function quotients in VA/VE

The Zipf law, along with the more popular Pareto analysis, is a fast method used for identifying a pattern in an apparently chaotic data volume[4]. In its elementary form, the Zipf law claims that if f is the frequency with which a certain phenomenon occurs and r is the rank (the number which indicates the position in the series of decreasing values of occurrence frequencies), then

$$f(r) = \frac{f_1}{r} \qquad (7)$$

in which f_1 is the frequency of rank 1 phenomenon (the highest occurrence frequency).

The American linguist and philologist George Kingsley Zipf(1902-1950) elaborated the first draft of the law after comparing the frequency of words in a considerable amount of written texts. Subsequently, analyzing for instance Corpus Brown(500 samples of texts in English, compiled from works published in the United States in 1961, with a total of one million words), it is noted that "the" is the word that shows up most frequently (69971 times), representing approximately 7% of all words. Confirming Zipf's Law, "of", the word ranked second, has just over 3,5% (36411 times), followed by "and" (28852 times) as shown in table 4.

Table 4. The most frequent words in Corpus Brown

Most frequent words	**the**	**of**	**and**
Frequency	7%	3,6%	2.8%
Rank	1	2	3
Frequency calculated with $f(r) = f_1/r$	7/1= 7%	7/2= 3,5%	7/3= 2,3%

It was also found that half of Corpus Brown used only 135 words [6].

In the Bible, the relation between the word frequency and their rank follows the empirical law (8) [5]

$$f(r) = \frac{a}{r^b} \qquad (8)$$

in which $a \approx 0{,}1$ and $b \approx 1$.

The Zipf law has a wide range of practical applications. For instance, there are seven limited liability companies on a given market and a new company needs to decid if it should enter the market. The largest of them being a limited liability company, its turnover and profit are public. According to the Zipf law, the second ranking company should have ½ of the first ranking company, the third one 1/3 and so on. The market size can be approximated by summing up the turnovers of the 7 companies. The overall market profit can be approximated in the same way. Knowing the market size, the market share of each company and their profit, it can be determined if the new company, the eighth one, with a turnover approximately equal to 1/8 of the leading company's turnover is sustainable[4].

The Zipf law is applicable for many other rankings and classifications unrelated to natural languages or the corporation size. For instance demographics(the population in cities from different countries, except for the former communist countries where the natural demographic processes have been significantly altered), the income distribution etc. In the case of city population, it has been noted that the correlation which best reflects this distribution is obtained for $b = 1,07$. It was also noted that, on large intervals and in a good approximation, a great deal of natural phenomena follow the Zipf law[6].

The Zipfs curves tend to get closer to the diagram axes when they are represented by means of linear scales[fig. 2]

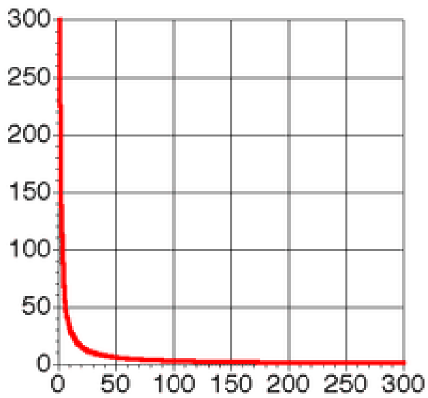

Fig. 2. Aspect of the Zipf distribution if linear scales are used on both axes

whereas displayed by means of a double-logarithmic diagram, they take the shape of a straight line[fig. 3].

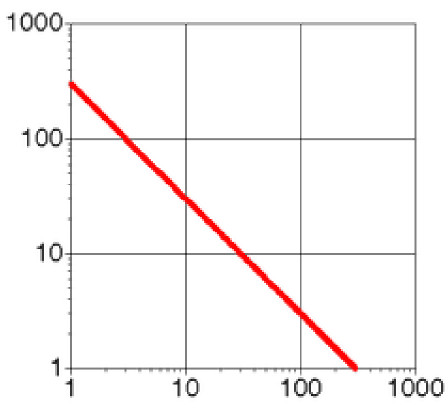

Fig. 3. Aspect of the Zipf distribution if logarithmic scales are used on both axes

A large number of modern studies confirm the precision of these somewhat strange irregularities in very different situations. In an attempt to apply the Zipf law to the information theory, Benoît Mandelbrot suggested a generalized version[5]:

$$f(r) = \frac{a}{(c+r)^b} \qquad (9)$$

a, b, and *c* being constants.

A simple description of data that fall within a Zipf distribution highlights the following:

- several elements have a very high score (the left side of the diagram)
- a small number of elements have a medium score (the middle part of the diagram)
- a very high number of elements have a very low score (the right side of the diagram)[7]

In other words, the Zipf distribution can be extracted from the Pareto distribution[1] by modifying the variables[6]. In memory of the three scientists aforementioned the distribution under discussion was named the Pareto-Zipf-Mandelbrot(PZM) distribution (law).

The different situations in which the PZM statistical distribution was discovered can be seen in [5] and [8]. Out of these, of particular interest are the size of Fortune 500 companies, all the languages spoken throughout the world and all times written texts, public transportation networks, airport networks, electric energy grids, the size of files stored on a PC and on the internet, the World Wide Web(the number of sites, links) the frequency of cited works in studies, the distribution of the tree trunks and branches, of blood vessels, lung structure.

Prices represent another area in which the Zipf law can be applied. The bell shaped Gaussian distribution is of no use in this case because the prices are not randomly distributed. The chart below indicates the price distribution for 172083 applications sold on the App Store, the Apple company online distribution system. Approximately 85000 applications(rank 1) cost 0.99 USD. The number of applications priced at 1.99 USD account for 1/4(rank 2) of the 0.99 USD applications. The 2.99USD applications represent 1/9 from the total(rank 3) and so on. In other words, the number of applications sold at a certain price is in inverse variation with the price(rank) squared value[9].

[1]The Pareto distribution was discovered over 100 years ago when Italian engineer, sociologist and economist Vilfredo Pareto (1848 – 1923) posed the problem of measuring differences between the rich and the poor. To achieve that, he collected data about fortune and income in different centuries and in different countries, noting that the data did not conform to the gaussian distribution (bell curve), as would have been expected should fortune and income have been alleatorily distributed. What he proceeded to discover was a very assymetric distribution of fortune and incomes. Starting from this truth, Pareto formulated the '80-20 law', according to which 20% of the population (the rich) control 80% of all income. Also, 20% of rich people own 80% of the 80% rich quota, and so on [10].

Fig. 4 – Prices following Zipf distribution Zipf [9]

Having reached this point, it is safe to assume that, just as very different fields, but especially the prices sector, fall within a PZM distribution, the function prices of at least some products, and respectively their costs and quotients, will in their turn, naturally follow the same distribution.

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