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COMPUTING THE INSTANTANEOUS FREQUENCY FOR AN ECG SIGNAL

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ABSTRACT

Physical phenomena have non-stationary nature that cannot be analyzed by conventional numerical methods. The Hilbert-Huang transform is a relatively new way to look at non-stationary signals by introducing the new concept of instantaneous frequency. This paper presents some remarks on how to compute the instantaneous frequency of a non-stationary signal, by the means of the Hilbert-Huang transform and with the empirical AM-FM decomposition.

Keywords: empirical mode decomposition, analytical signal, Hilbert transform, AM-FM decomposition, instantaneous frequency

1. Introduction

Until recent years the concept of frequency had a definition based on the Fourier theorem. But as several studies had shown [1], the Fourier transform can be applied only to a handful of signals, that are linear, periodic or stationary; otherwise, the resulting spectrum rarely makes physical sense. In most cases data retrieved from physical systems will not meet the conditions needed by the Fourier transform.

The method we have studied is based on the empirical mode decomposition (EMD) which generates a set of intrinsic mode functions (IMF) from which an "instantaneous frequency" can be computed. This is practical for nonlinear and non-stationary signals. The notion of the instantaneous frequency has been the base of many discussions. There are two problems in understanding the idea behind the instantaneous frequency: the first one has its roots in the Fourier analysis, which describes signals as sums of sine and cosine waves, existing through the whole time span of the data. In non-stationary signals a constant frequency wouldn't make sense. The other problem is represented by the fact that instantaneous frequency cannot be defined in a unique way.

One of the methods capable of computing the instantaneous frequency is the Hilbert-Huang transform, but this has quite severe limitations with respect to the correctness of the given result. Another approach, the empirical AM-FM decomposition, on the other hand, overcomes these limitations as will be seen in following sections.

2. The empirical mode decomposition

Signals can be decomposed in many ways by applying different mathematical transformations on them. Well known examples are the Wavelet transform and the Fourier transform. Generally these transforms are used to offer information about the frequency data contained by the signal. A relatively new idea is to find a signal's instantaneous frequency, which is somewhat different from the frequency concept known from the Fourier analysis. The instantaneous frequency can only be calculated for monocomponent signals or intrinsic mode functions [1]. As such, a monocomponent (or IMF) is defined as a signal that has the number of local maxima equal to the number of local minima or differ at most by one and the upper and lower envelopes of the signal are symmetric with respect to zero.

The empirical mode decomposition is a method that decomposes any signal into intrinsic mode functions. There is no analytical definition for this method, for it is described by an iterative algorithm.

Upper and lower envelopes are constructed for the original signal with the help of cubic spline approximation and their mean is computed. Then, the mean of the envelopes is subtracted from the original signal. The process of extracting an IMF from the signal is called sifting [5], [7], [10], [12]. The sifting stops when a sum of differences is smaller than a predefined value. This is like a convergence test and it was developed by Huang [1]:

$$SD_k = \frac{\sum_{t=0}^{T} |h_{k-1}(t) - h_k(t)|^2}{\sum_{t=0}^{T} h_{k-1}^2(t)} \quad (1)$$

where h_{k-1} and h_k are consecutive approximations of the IMF. The predefined value for this sum of differences was chosen as 0.2. Once the sifting process is done, the resulting IMF is subtracted from the signal and the whole process is repeated with the remaining residue until this residue is a monotonic function [1], [2].

3. Computing the instantaneous frequency with the Hilbert transform

Once the IMFs of a signal are calculated with the empirical mode decomposition, a corresponding analytical signal can be constructed for each of them using the Hilbert transform. There are a number of conditions imposed by the Bedrosian and Nuttall theorems [4], [8] for a signal to be Hilbert transformable. An IMF that results from the EMD does not satisfy all these conditions. However, the erroneous behavior is not always significant, or it is not present in many cases. Equation (2) presents the form of the analytical signal of an IMF

$$z(t) = x(t) + iy(t) = a(t)e^{i\varphi(t)} \quad (2)$$

where $y(t)$ is the Hilbert transform of the IMF $x(t)$:

$$y(t) = \frac{1}{\pi} P.V. \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau \quad (3)$$

Having the analytical signal of an IMF, the instantaneous phase and instantaneous frequency can be computed as:

$$\varphi(t) = atan \frac{y(t)}{x(t)} \quad (4)$$

$$\omega(t) = \frac{d\varphi(t)}{dt} \quad (5)$$

The instantaneous amplitude is:

$$a(t) = \sqrt{x^2(t) + y^2(t)} \quad (6)$$

4. Computing the instantaneous frequency with the empirical AM-FM decomposition

Because of the limitations presented in the previous section, new methods had been developed to compute the instantaneous frequency. Such a method is the empirical AM and FM decomposition.

AM and FM decomposition may be applied to IMF signals and it follows a normalization pattern. The separation of the AM and the FM component of an IMF can be realized uniquely by an iterative algorithm. Thus, a cubic spline is used to approximate the absolute value of the IMF. Using this envelope, $e_1(t)$, the original IMF is normalized:

$$y_1(t) = \frac{x(t)}{e_1(t)} \quad (7)$$

The result is then approximated again with a cubic spline and the normalization goes on:

$$y_n(t) = \frac{y_{n-1}(t)}{e_n(t)} \quad (8)$$

until $y_n(t)$ has its values in the interval $[-1;\ 1]$. This final $y_n(t)$ represents the FM part of the original IMF. The AM part is then computed by:

$$A(t) = \frac{x(t)}{F(t)} \quad (9)$$

where $A(t)$ is the AM part of the IMF and $F(t)$ is the FM part of the IMF.

As the original signal can be approximated by:

$$x(t) = A(t)cos\varphi(t), \quad (10)$$

in [1] has been shown that $A(t)$ is the AM part of the signal, while $cos\varphi(t)$ represents the FM part. Considering FM to be a cosine-like signal, we can calculate its quadrature, $Q(t)$, so that [2]

$$F^2(t) + Q^2(t) = 1 \quad (11)$$

This leads us to the fact that $Q(t)$ is a sine-like signal. Using these two signals, i.e. the FM signal and its quadrature, we are able to compute the instantaneous phase of the original IMF:

$$\varphi(t) = atan \frac{Q(t)}{F(t)} \quad (12)$$

The inverse tangent function gives information about both the value of the angle and the quadrant in which it is situated, so the phase is fully defined. After unwrapping the phase, the instantaneous frequency can be found using:

$$\omega(t) = \frac{d\varphi(t)}{dt} \quad (13)$$

5. Results

The presented methods were applied to an ECG signal selected from the MIT-BIH arrhythmia database, as shown on fig. 1.

The result of the empirical mode decomposition provided us six intrinsic mode functions (fig. 2), on which both the Hilbert transform and AM-FM decomposition had been applied.

Presented on fig. 3 is the second IMF and its respective instantaneous phase and instantaneous frequency computed with the help of the Hilbert transform. Figure 4 shows the results obtained with the AM-FM decomposition. Although the two methods refer to the instantaneous frequency and phase, the results are different. The AM-FM method offers more detailed information about frequency taking into account every little change in the IMF, regardless of its amplitude. The spikes that are present in both figures are the side effect of the derivative used to compute the instantaneous frequency. By smoothing the instantaneous phase, one can get a “cleaner” result. Figure 5 presents the 3D representation of the time, instantaneous frequency and amplitude of the second IMF.

This representation facilitates the discovery of hidden patterns and energy distribution through the IMF. This can be useful to differentiate real life signals from computer-generated ones.

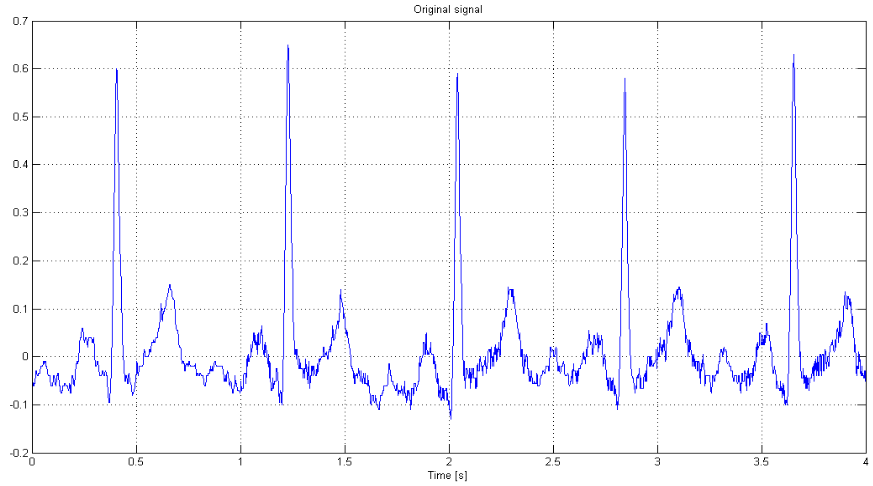

Fig. 1 The original signal

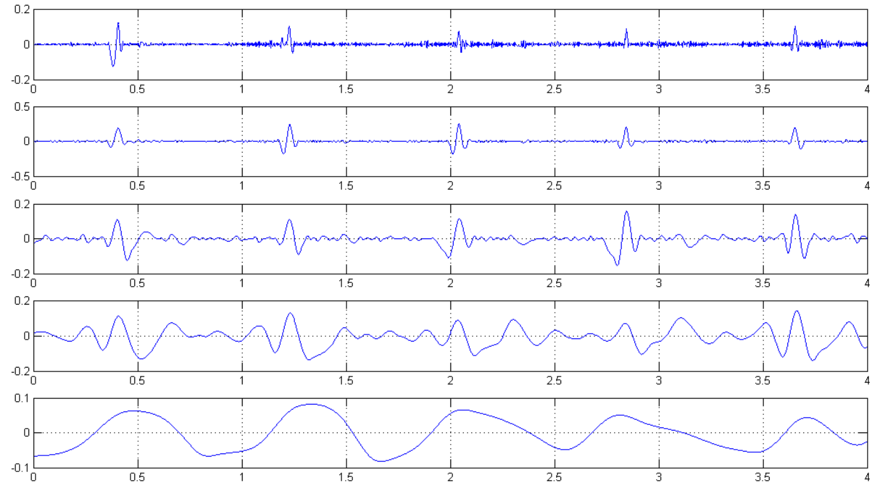

Fig. 2 The intrinsic mode functions of the original signal

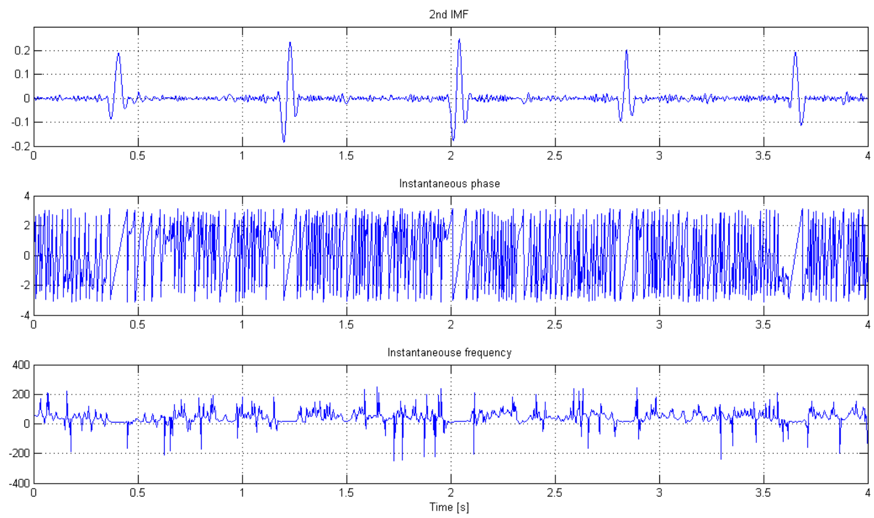

Fig. 3 The 2nd IMF and its instantaneous phase and instantaneous frequency as computed with the Hilbert transform

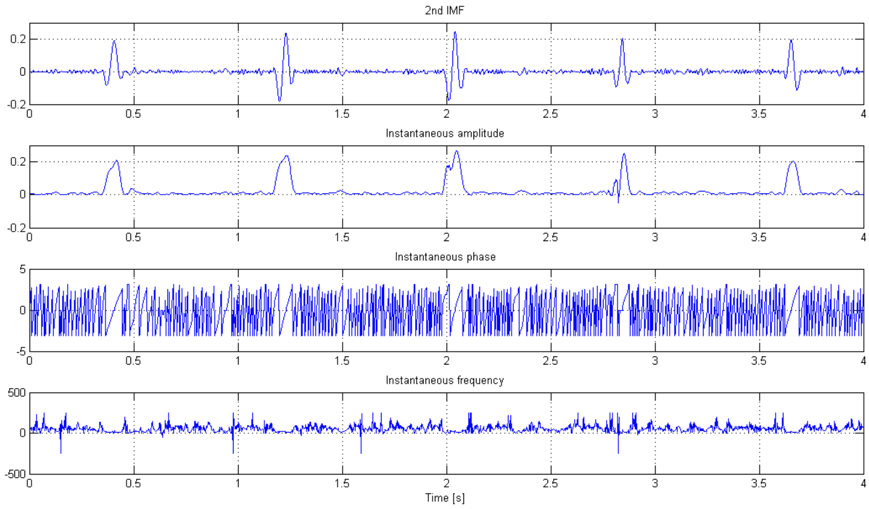

Fig. 4 The 2nd IMF and its instantaneous amplitude, instantaneous phase and instantaneous frequency as computed with the AM-FM decomposition

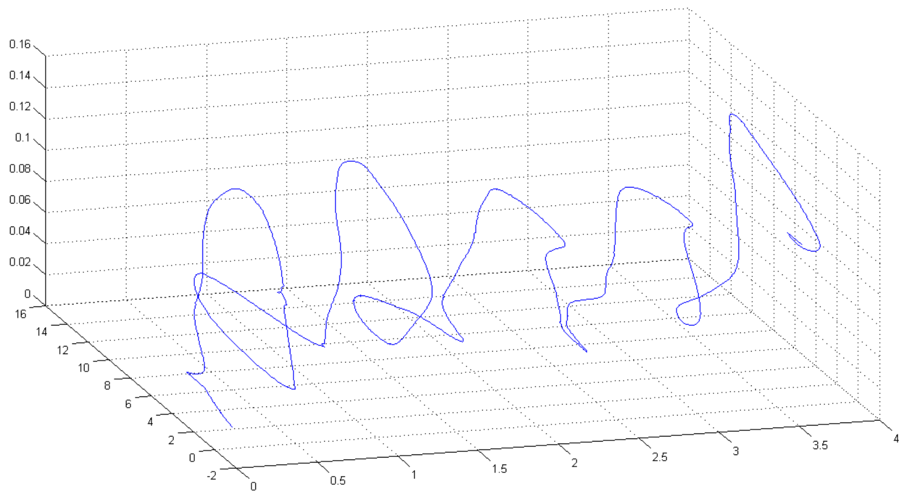

Fig. 5 The joint representation of time, instantaneous frequency and amplitude of the 2nd IMF computed with the AM-FM decomposition

6. Conclusions

The frequency in Hilbert spectrum gives another meaning to the concept of "frequency" widely known from the Fourier theory. The Fourier representation of a frequency ω, means a component of a sine wave being present through the whole data. One frequency component ω means only that, in the signal it has appeared at some location.

On the other hand, the Hilbert spectrum represents a joint amplitude – frequency - time distribution. The DC term of Fourier spectrum is present because of the non-zero mean, concentrating a considerable amount of energy into a single point, while the marginal spectrum of an IMF gives a continuous distribution of energy through the time span of the data. Thus, the Fourier spectrum offers no meaningful information if the data is not stationary.

Even the Hilbert transform can't provide always a meaningful instantaneous frequency if the Fourier transform of the analyzed signal doesn't have only positive frequency values.

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