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VIBRATION MODELLING FOR SYSTEMS WITH COMPLEX STRUCTURE

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Abstract
This paper presents vibration model and vibration analysis of a mechanical system with two components. Each mechanical system has a resonant frequency and if the external force which acts on the system has the same frequency, the vibration caused by the force leads to destruction. Mechanical systems have many components which are fixed together in different ways and react in different manner to an external force. The question is how reacts a system composed of two different masses to a sinusoidal base vibration. The article presents a possible model of a frame. The frame is modeled with two concentrated masses. The moving equations are developed for this model, than the movements of the masses and of the model are studied in different cases, different vibration frequencies, different stiffness and masses. For the study Matlab simulation was used. The resonant case was also studied. The simulation result shows that the stability of the frame depends on the number of the columns, the frequency of the vibration and on the masses.

Keywords: accelerometer, frame model, vibration modeling, vibration analysis, Matlab simulation

1. Introduction

Vibration and shock are present in all areas of our lives. Vibrations are mechanical oscillations around of a fixed point and define the movement of a mechanical system. Vibrations can be characterized in many ways, there are vibrations with low or high frequencies, and there are unintended vibrations (perturbations) or generated vibrations with known parameters. They may be generated and transmitted by motors, turbines, machine-tools, bridges, towers, and even by the human body. Consequently, there is often a need to understand the causes of vibrations and to develop methods to measure and prevent them.

To analyze the vibrations and the effects produced by these it will be measured the vibration in more points of the system: at the output of the system to compare the measured values with the maximum admissible values, this is the case of noise and perturbation detection. The characteristics of the vibration can be measured at the input of the system to set these at the established values. To test the system how reacts to the different external forces and noises, vibrations must be measured both at the input and the output of the system.

A mechanical system with many components vibrates oscillates during his operating time and this can be in most time unintended and leads to breakdowns, for example vibrating motors or belts caused by inadequately fixing [3], [4].

Components of mechanical systems are fixed together more or less stark or flexible. If the mechanical system vibrates these components vibrates also, but in different ways because the acting force, the mass, the damping coefficient, the springiness coefficient, the resonance pulse are different for each component.

A system composed of two masses, two springs and two dampers has a vibration reply composed of the answer of the two components. The amplitude-frequency has two peaks proper to the components. Figure shows a system with two components and the suitable amplitude and phase characteristics [2].

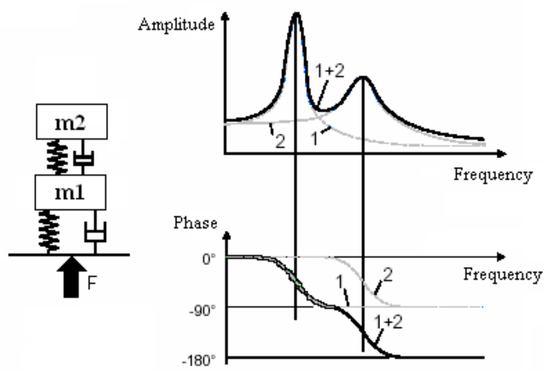

Fig.1: The amplitude and phase characteristic for a system with two degrees of freedom.

A real mechanical system is more complicated, the components can move in more than one direction and the vibration spectrum has many peaks suitable for each pieces. A real mechanical system is a system with many degrees of freedom. In certain cases even simple system can be studied as a system with two or more degrees of freedom, like motors fixed to the base through bearings.

2. The cause of the vibration

Applying an external sinusoidal force to a mechanical system this will move with the same frequency as the force. A real mechanical system is composed on many pieces which are bound in different ways and react in different manner to the force and this differences cause repeating force apparition. Repeating forces are due to the rotation of imbalanced or misaligned components. Imbalance is caused by corroded, deformed, broken parts, gaps, non-uniform material density, and component sizes variation [6], [7].

Misalignments are caused by inaccurate mounting, distortions, bad assembly. Wrong pieces cause also force apparition and undesirable vibrations.

If the frequency of the force is near to the natural oscillation rate of the system this will vibrate more and more strongly and brings the system in resonance. Repeating force would not be a problem until it begins to cause resonance. Resonance should always be avoided because determines in very short time severe damages [5].

3. Vibration analysis

Vibrations are mechanical oscillations and so they can be characterized with amplitude and frequency. Amplitude shows how strongly the vibration is, and frequency shows the oscillation rate of vibration. These two provide information to identify the root of vibration.

The amplitude is related to the speed of the movement and to the force which cause the vibration. The peak of the amplitude shows the highest speed and the rms value of the amplitude shows the vibration energy. The frequency is related to the condition of the system.

Vibration analysis consists of a spectral analysis. The spectrum is a very useful analytical tool because shows the frequencies at which vibration occurs. The information from a spectrum depends on the maximum frequency F_{max} and resolution. How high F_{max} needs to be is dependent on the operating speed of the system. The resolution of a spectrum establishes the detail in the spectrum. [3], [10]

For vibration measurements the preferred sensor is the accelerometer. This generates an output signal that is proportional to the acceleration of the vibrating system. Accelerometers have good sensitivity, wide frequency range; they are small and light in weight and are capable to measuring the vibration at a specific point. It can be used electronic networks to obtain a voltage proportional to velocity or displacement [9].

To detect true vibration behavior of a system the accelerometer needs to undergo exactly the same vibratory movement as the vibrating system. An accelerometer must therefore be attached firmly to the vibrating pieces [1], [3].

By vibration measurement several spectra are measured and then an average spectrum is produced. The average spectrum better represents vibration behavior. The measurement system used for diagnostic analyses of vibrations consists of the following components shown in figure 2 [2].

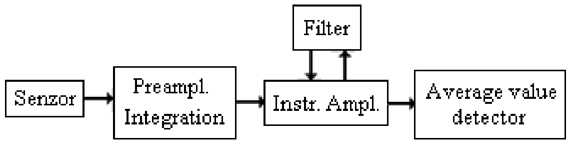

Fig.2: Vibration measurement system.

4. System vibration modeling

A mechanical system can be modeled with a mass, main spring and damper. If an external force acts on a mechanical system, this will have a movement with the same frequency as the force. If an external force acts on a mechanical system composed on two masses, main springs and dampers the system response will be a combination of the two simple system response. On the amplitude-frequency plot appears two peaks proper for the two components. Mechanical systems with single degree of freedom can move in a single direction. If they can move in two directions they have two degree of freedom.

Systems with two degrees of freedom can be for example a multi-level structure, frame or a tower. These systems can be ideally represented by an assemblage made of elements connected among them. Such systems have many degrees of freedom, but fixing the elements and allow the movement in certain directions, systems with a single or two degrees of freedom can be obtained. The whole interconnected elements can be reduced to concentrated masses and columns. For a two level structure, frame mass concentration is shown on figure 3a, if the body is rough. Columns can be represented as springs with stiffness *k* and unstretched length. The damping does not exist in this case [11], [12].

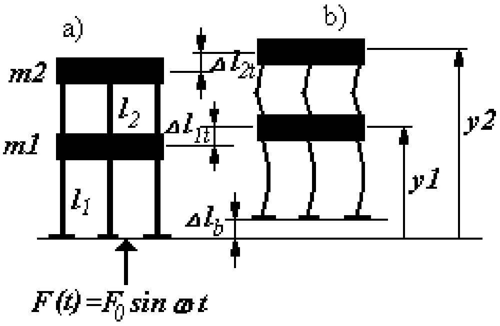

Fig.3: Schematic representation for a frame and his movement.

On this system acts a vertical force with sinusoidal form, $F(t)=F_0 sin\omega t$. The base will have a sinusoidal movement with a maximum displacement d_0. Concentrated masses will move also vertically, figure 3b, and the columns are changing theirs lengths. The total mass displacement will be given by the sum of the relative displacement between the base and the masses and the displacement of the base [11].

$$\Delta l_{it} = \Delta l_i + \Delta l_b \quad i=1,2 \tag{1}$$

The main problem is to estimate the vibration, the movement of the two masses [8].

At time *t* the positions of the masses from a fixed point are *y1* and *y2*. The first and the second derivate of the position give the velocity and the acceleration of the masses.

$$v_1 = \frac{dy_1}{dt} \qquad a_1 = \frac{d^2y_1}{dt^2}$$
$$v_2 = \frac{dy_2}{dt} \qquad a_2 = \frac{d^2y_2}{dt^2} \tag{2}$$

The accelerometers measure the accelerations of the masses. [8]

For the structure presented on figure 3, on the masses are acting resultant forces given by the following equations:

$$F_2 = -m_2 g - 3F_{c2}$$
$$F_1 = -m_1 g - 3F_{c1} + 3F_{c2} \tag{3}$$

Where F_{c1} and F_{c2} are the forces in columns (springs) under tension. These forces in springs are proportional with the modification in length of the springs.

$$F_{c1} = k\Delta l_1 \qquad F_{c2} = k\Delta l_2 \tag{4}$$

The columns lengths depend on initial length *L*, on weight of the mass and on the base position at *t* moment when the base is vibrating. Initial the columns lengths can be written as:

$$l_1 = L - \frac{m_1 g}{k} \qquad l_2 = L - \frac{m_2 g}{2k} \tag{5}$$

At moment *t* if the base is vibrating and has a movement described with $d(t)=d_0 sin\omega t$ the columns lengths can be written:

$$l_1 = y_1 - d_0 sin\omega t \qquad l_2 = y_2 - y_1 \tag{6}$$

The acting forces are:

$$F_1 = -m_1 g - 3k(y_1 - L - d_0 sin\omega t) + 3k(y_2 - y_1 - L)$$

$$F_2 = -m_2 g - 3k(y_2 - y_1 - L) \tag{7}$$

The movement of the masses are given by the total force, written as a function of acceleration; $\vec{F} = m\vec{a}$ (Newton's laws). The accelerations are given by the relations (2). Made the substitutions the two equations of the motions are:

$$\frac{d^2y_1}{dt^2} = -g - \frac{3k}{m_1}(y_1 - L - d_0 sin\omega t) + \frac{3k}{m_1}(y_2 - y_1 - L)$$

$$\frac{d^2y_2}{dt^2} = -g - \frac{3k}{m_2}(y_2 - y_1 - L) \tag{8}$$

To solve the problem we need to identify the initial conditions, the positions of the masses from the fixed point, at time *t=0*:

$$y_1 = L - \frac{m_1 g}{k} \qquad \frac{dy_1}{dt} = 0$$

$$y_2 = 2L - \frac{3m_2 g}{2k} \qquad \frac{dy_2}{dt} = 0 \tag{9}$$

To study the position variation in time the equations of motions must be expressed in terms of the first derivations, introducing new variables, the velocities of the masses.

$$\frac{dy_1}{dt} = v_1$$
$$\frac{dy_2}{dt} = v_2$$
$$\frac{dv_1}{dt} = -g - \frac{3k}{m_1}(y_1 - L - d_0 sin\omega t) + \frac{3k}{m_1}(y_2 - y_1 - L)$$

$$\frac{dv_2}{dt} = -g - \frac{3k}{m_2}(y_2 - y_1 - L) \tag{10}$$

This equation of motion was simulated in Matlab in different conditions.

Figure 4 shows the vibration of the system for vibration frequency smaller then the system's frequency if the masses are equal and figure 5 for vibration frequency around the resonant frequency.

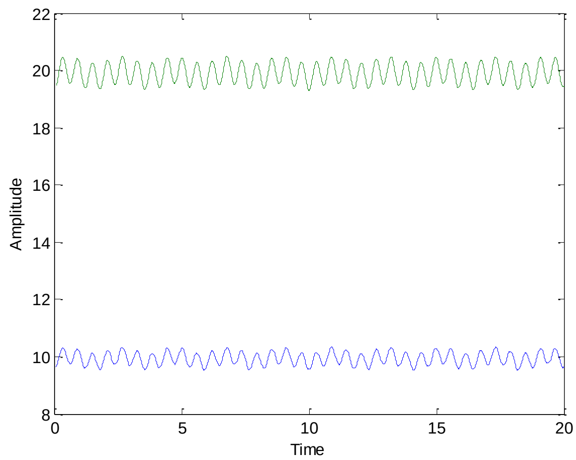

Fig. 4: System vibration for k=100,ω=3,m1=m2.

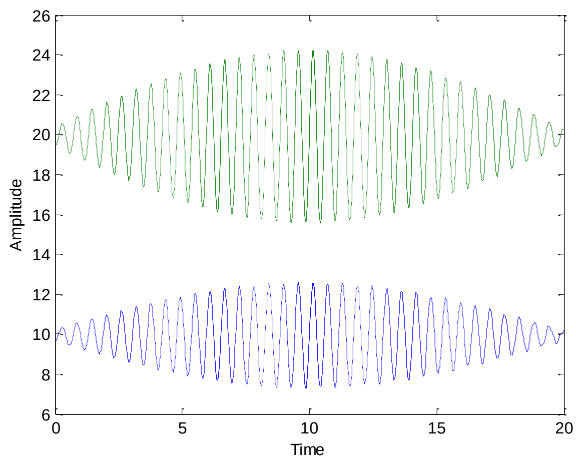

Fig. 5: System vibration for k=100,ω=11,m1=m2.

It can be observed the amplitude of the mass vibration increase around the resonant frequency.

Increasing the m2 the vibration amplitude is decreasing, figure 6.

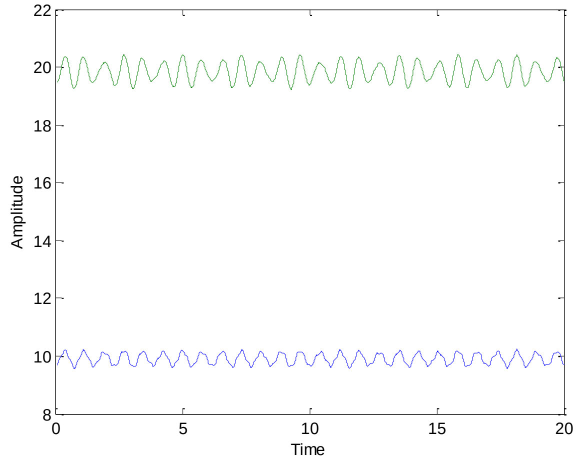

Fig.6. System vibration for *k=100,ω=11,m1=1,m2=2*.

The initial length of the columns does not influence the system vibration.

Changing the number of columns from three to two the stability of the system is smaller. In this case the equation of motion is changing a little. Figure 7 and figure 8 shows the system vibration for different vibration frequency when the fame has two columns [8].

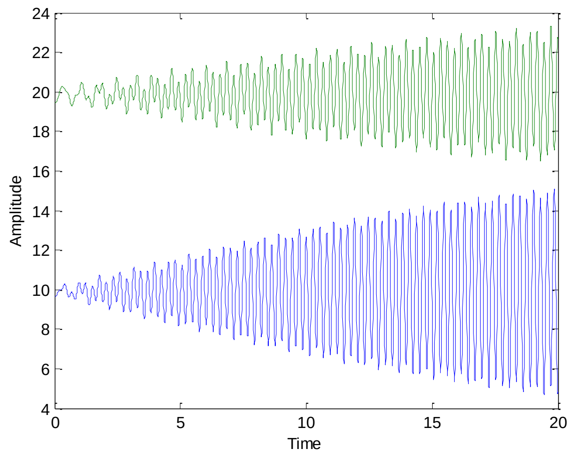

Fig.7. System vibration for k=100,ω=23,m1=m2.

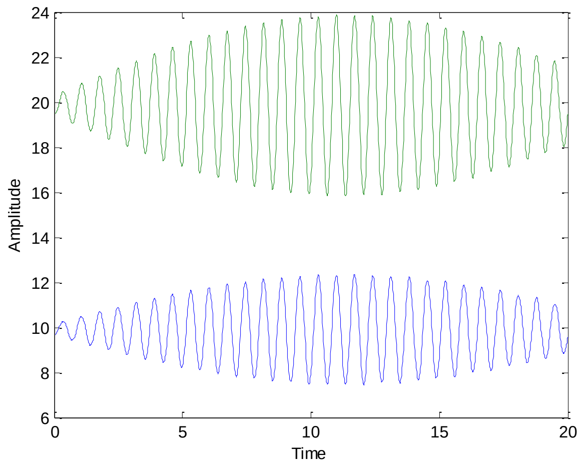

Fig.8: System vibration for k=100,ω=9,m1=m2.

The concentrated masses of the system are bound with the columns and have an influence of each other. The vibration of the system depends on the sum of the vibration of the masses. Figure 9 shows the resulted vibration.

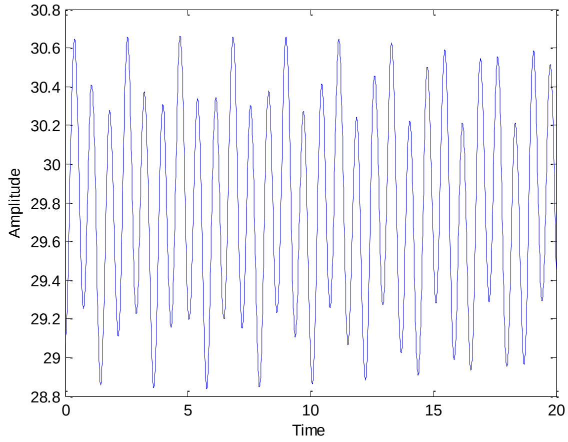

Fig.9: Result system vibration for k=100,ω=3.

To study mechanical systems behavior at vibrations a schematic representation can be made. In this representation the main component can be concentrated in masses which are connected through stiffness coefficient and damping coefficient. The values of these coefficients depend on the type and material of the connection.

Studying systems behavior based on schematic representation gives information regarding at modification through which the stability of the system can be increase or the resonant frequency of the system can be change to avoid the damages.

4. Conclusions

If a mechanical system vibrates his components vibrates also, but in different ways because the acting force, the mass, the damping coefficient, the springiness coefficient, the resonance pulse are different for each component.

A schematic representation for a two level structure, frame was made. It was estimate the vibration, the movement of the two concentrated masses of the frame caused by an external force.

The equation of the motion was deduced and simulated in Matlab for different cases. It can be observed the stability of the frame depends on the number of the columns, the frequency of the vibration and on the masses. The resonant frequency of the system must be avoided.

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