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EXTENSION OF HOMOTOPY DECOMPOSITION METHOD (HDM) TO COUPLED NONLINEAR VAN DER POL TYPE'S EQUATION

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Abstract

We put into operation a moderately original analytical modus operandi, the Homotopy decomposition method, for getting to the bottom of coupled nonlinear van der Pol type's equations arising in chaotic synchronization dynamics of coupled non-linear oscillators. Numerical solutions are given, and some properties show signs of physically reasonable reliance on the parameter values. The trustworthiness of HDM and the lessening in calculation bestow HDM a wider applicability. On top, the calculations concerned in HDM are especially trouble-free and undemanding. It is verified that HDM is a controlling and capable utensil for nonlinear ODEs. It was also established that HDM is further professional than the PEM, ADM, VIM, HAM and HPM. We can without difficulty bring to a close that the Homotopy decomposition method is an efficient tool to solve approximate solution of nonlinear partial differential equations.

Keywords: Homotopy decomposition method, coupled Nonlinear van der Pol Type's equation, complexity.

1. Introduction

In current duration, wide-ranging of exploration have been carried out to analyze disordered harmonization dynamics of coupled non-linear oscillators [1-2]. The attention dedicated to such topics is outstanding to the potential applications of harmonization in communication engineering using pandemonium to mask the information signals [3-4], electrical and automation engineering, biology, and chemistry [5-6].

Van der Pol equation makes available an illustration of an oscillator with nonlinear damping, energy being debauched at great amplitudes and produced at near to the ground amplitudes. Such systems characteristically have power over limit cycles: continued oscillations around a state at which energy generation and dissipation balance [7]. The innovative appliance depicted by van der Pol models an electrical circuit with a triode valve, the resistive properties of which change with current, the low current, negative resistance becoming positive as current increases [7]. This model has been extensively applied in science and engineering [8-12]. Presently, Nohara and Arimoto measured the oscillation system of two linearly coupled van der Pol equations [13]. They demonstrated that there exist either in-phase or out-of-phase periodic solutions only.

In this paper, we further investigate the existence of solutions of two nonlinearly coupled van der Pol equations by means of the decomposition method HDM [14]. Unlike perturbation techniques [15-17], the HDM is independent of any small physical parameters and the Abel integral. More important, the HDM provides a simple way to ensure the convergence of solution series so that one can always get accurate enough approximations even for strongly nonlinear problems [22-23]. The method was first used to solve the groundwater flow equation. To show the efficiency of the method, for solving the coupled van der Pol oscillation equation, the following three problems are considered.

Problem 1: Let us consider the periodic solutions of the two coupled van der Pol equations,

$$\begin{cases} \ddot{y}(x) + \varepsilon(y^2 - 1)\dot{y}(x) + y(x) = k\big(\big(y(x) - z(x)\big) + \mu(y^3(x) - z^3(x)\big) \\ \ddot{z}(x) + \varepsilon(z^2 - 1)\dot{z}(x) + z(x) = k\left(z(x) - y(x)\right) + \mu(z^3(x) - y^3(x)) \end{cases}$$

subjected to the initials conditions:
$y(0) = \alpha, \dot{y}(0) = 0, z(0) = c\alpha, and\ \dot{z}(0) = \alpha\omega d.$

The details on parameters are specified later in Section 4.

Problem 2: Consider the following model of a classical van der Pol oscillator coupled gyroscopically to a linear oscillator

$$\begin{cases} \ddot{y}(t) + \varepsilon(y^2 - 1)\dot{y}(t) + y(t) + h\dot{x}(t) = E\cos(nt) \\ \ddot{x}(t) + \lambda\dot{x}(t) + x(t) - dy(t) = 0. \end{cases}$$

Problem 3: Let us consider the periodic solutions of the two coupled van der Pol equations

$$\begin{cases} \ddot{x}(t) + x(t) - \mu\left(1 - x^2(t)\right)\dot{x}(t) + \alpha x^3(t) = E_0\cos(t) \\ \ddot{y}(t) + (1+K)y(t) - \mu\left(1 - y^2(t)\right)\dot{y}(t) + \alpha y^3(t) - Kx(t) = E_0\cos(\varepsilon t). \end{cases}$$

The paper is prearranged as tag along: In Section 2, we show the indispensable proposal of the homotopy decomposition method for getting to the bottom of towering orders differential equations. We show the appliance of the HDM for coupled van der Pol type of equations and numerical results in Section 3. In order to simplicity of this method, in Section 4 we show the intricacy of the method for coupled van der Pol's types of equations. The conclusions are then given in the final Section 5.

2. Basic apparatus about the homotopy decomposition method

To exemplify the indispensable suggestion of this process we consider a general nonlinear non-homogeneous differential equation of the following form

$$\frac{\partial^m B(x)}{\partial x^m} = L\left(B(x)\right) + N\left(B(x)\right) + f(x), \quad m = 1,2,3 \ldots \tag{1}$$

subject to the initial conditions

$$\frac{\partial^i B(0)}{\partial x^i} = y_i, \ \frac{\partial^{m-1} B(0)}{\partial x^{m-1}} = 0\ , i = 0,1,2 \ldots m-2,$$

where f is a known function, N is the general nonlinear differential operator and L represents a linear differential operator. The technique foremost movement here is to affect the inverse operator $\frac{\partial^m}{\partial x^m}$ of on both sides of equation (1) to obtain,

$$B(x) = \sum_{k=0}^{m-1} \frac{x^i}{i!} y_i + \int_0^x \int_0^{x_1} .. \int_0^{x_{m-1}} L\left(B(\tau)\right) + N\left(B(\tau)\right) + f(\tau)\, d\tau .. dx \tag{2}$$

The multi-integral in equation (2) can be distorted to

$$\int_0^x \int_0^{x_1} .. \int_0^{x_{m-1}} L\left(B(\tau)\right) + N\left(B(\tau)\right) + f(\tau) d\tau .. dx = \frac{1}{(m-1)!} \int_0^x (x-\tau)^{m-1} L\left(B(\tau)\right) + N\left(B(\tau)\right) + f(\tau) d\tau,$$

so that equation (2) can be reformulated as

$$B(x) = \sum_{i=0}^{m-1} \frac{x^i y_i}{i!} + \frac{1}{(m-1)!} \int_0^x (x-\tau)^{m-1} L\left(B(\tau)\right) + N\left(B(\tau)\right) + f(\tau)\, d\tau.$$

Using the Homotopy design method, the solution of the above integral equation is given in series form as:

$$B(x,p) = \sum_{n=0}^{\infty} p^n B_n(x)$$
$$B(x) = \lim_{p \to 1} B(x,p)$$

and the nonlinear term can be decomposed as

$$NB(x) = \sum_{n=1}^{\infty} p^n \mathcal{H}_n(B)$$

everywhere $p \epsilon\ (0,\ 1]$ is an embedding parameter and $\mathcal{H}_n(B)$ is the He's polynomials [20] that can be generated by

$$\mathcal{H}_n(B_0, \cdots\cdots, B_n) = \frac{1}{n!}\frac{\partial^n}{\partial p^n}\left[N\left(\sum_{j=0}^{n} p^j B_j(x)\right)\right], n = 0,1,2 \cdots$$

The homotopy decomposition method is got hold of by the beautiful combination of decomposition method with Abel integral

$$\sum_{n=0}^{\infty} p^n B_n(x) = T(x) + p\frac{1}{(m-1)!}\int_0^x (x-\tau)^{m-1}[f(\tau) + L(\sum_{n=0}^{\infty} p^n B_n(\tau)) + \sum_{n=0}^{\infty} p^n \mathcal{H}_n(B)]d\tau$$

with $T(x) = \sum_{i=0}^{m-1} \frac{x^i}{i!}$ weigh against the vocabulary of identical powers of *p*, furnish solutions of an assortment of orders. The preliminary deduction of the rough calculation is $T(x)$. The convergence of the method can be found in [14].

Remark 1: The initial guess $T(x)$ is the Taylor series of the exact solution of order m

3. Complexity of HDM

It is very important to test the computational complexity of the method or algorithm [14]. We compute a coupled van der Pol type's equation example which is solved by the homotopy decomposition method. The code has been presented with Mathematica 8 according to the following pseudocode.

Step 1: Set $m \leftarrow 0$

Step 2: Calculated the recursive relation after the comparison of the terms of the same power is done.

Step 3: If $\|x_{n+1}(t) - x_n(t)\| < r$, $\|y_{n+1}(t) - y_n(t)\| < r$ with r the ratio of the neighbourhood of the exact solution then go to step 4, else $m \leftarrow m+1$ and go to step 2

Step 4: Print out

$x(t) = \sum_{n=0}^{\infty} x_n(t)$, $y(t) = \sum_{n=0}^{\infty} y_n(t)$

as the approximate of the exact solution.

Lemma 1: *The complexity of the homotopy decomposition method for solving coupled nonlinear equation is of order* $O(n)$.

Proof: The number of computations including product, subtraction and division are in step 2

x_0, y_0: 0 because, obtains directly from the initial guess

(x_1, y_1) : (3, 3)

⋮.

(x_n, y_n): (3, 3)

Now in step 4 the total number of computations is equal to $\sum_{j=0}^{n} x_j(t) = 3n = O(n)$ and $\sum_{j=0}^{n} y_j(t) = 3n = O(n)$.

4. Application

In this section we apply this method for solving van der Pol oscillator coupled equations.

Problem 1: Let us consider the periodic solutions of the two coupled van der Pol equations,

$$\begin{cases} \ddot{y}(x) + \varepsilon(y^2-1)\dot{y}(x) + y(x) = k\big(\big(y(x) - z(x)\big) + \mu(y^3(x) - z^3(x)\big) \\ \ddot{z}(x) + \varepsilon(z^2-1)\dot{z}(x) + z(x) = k\big(z(x) - y(x)\big) + \mu\big(z^3(x) - y^3(x)\big) \end{cases}$$

subjected to the initials conditions:

$y(0) = \alpha, \dot{y}(0) = 0, z(0) = c\alpha,$ and $\dot{z}(0) = \alpha\omega d$

where ε is a physical parameter for typical van der Pol oscillations, k and μ are the linear and nonlinear couple parameters, respectively [7]. It is well known that free oscillations of self-excited systems have periodic limit cycles independent of initial conditions. Periodic solutions of self-exited systems contain two important physical parameters, i.e., the angular frequency ω and the amplitude α [7].

Subsequent the HDM steps, we obtained the following:

$$\sum_{n=0}^{\infty} p^n y_n(x) - T(x)$$
$$= p\int_0^x (x-t)\left(-\varepsilon\left(\left(\sum_{n=0}^{\infty} p^n y_n(t)\right)^2 - 1\right)\sum_{n=0}^{\infty} p^n \dot{y}_n(t) - \sum_{n=0}^{\infty} p^n y_n(t)\right.$$
$$\left. + k\left(\left(\sum_{n=0}^{\infty} p^n y_n(t) - \sum_{n=0}^{\infty} p^n z_n(t)\right) + \mu\left(\left(\sum_{n=0}^{\infty} p^n y_n(t)\right)^3 - \left(\sum_{n=0}^{\infty} p^n z_n(t)\right)^3\right)\right)\right)$$

$$\sum_{n=0}^{\infty} p^n z_n(x) - f(x)$$
$$= p\int_0^x (x-t)\left(-\varepsilon\left(\left(\sum_{n=0}^{\infty} p^n z_n(t)\right)^2 - 1\right)\sum_{n=0}^{\infty} p^n \dot{y}_n(t) - \sum_{n=0}^{\infty} p^n y_n(t)\right) dt$$
$$+ k\left(\left(\sum_{n=0}^{\infty} p^n y_n(t) - \sum_{n=0}^{\infty} p^n z_n(t)\right) + \mu\left(\left(\sum_{n=0}^{\infty} p^n y_n(t)\right)^3 - \left(\sum_{n=0}^{\infty} p^n z_n(t)\right)^3\right)\right)$$

Comparing the terms of the same power of p, we obtain the following systems of integral equations:

$$p^0: y_0(x) = T(x),\ y_0(x) = \alpha$$

$$p^0: z_0(x) = f(x), \qquad z_0(x) = c\alpha$$

$$p^1: y_1(x) = \int_0^x (x-t)[-\varepsilon(y_0^2 - 1)\dot{y}_0 - y_0 + k(y_0 - z_0) + \mu(y_0^3 - z_0^3)]dt, y_1(0) = 0$$

$$p^1: z_1(x) = \int_0^x (x-t)[-\varepsilon(z_0^2 - 1)\dot{z}_0 - z_0 + k(z_0 - y_0) + \mu(z_0^3 - y_0^3)]dt,\ z_1(0) = 0$$

⋮

$$p^n: y_n(x) = \int_0^x (x-t)\left[-\varepsilon\left(\sum_{j=0}^{n-1} y_j\, y_{n-j-1}\dot{y}_{n-j-1} - \dot{y}_{n-1}\right) - y_{n-1} + k(y_{n-1} - z_{n-1})\right.$$
$$\left. + \mu\left(\sum_{j=0}^{n-1}\sum_{k=0}^{j} y_j\, y_{j-k} y_{n-j-1} - \sum_{j=0}^{n-1}\sum_{k=0}^{j} z_j\, z_{j-k} z_{n-j-1}\right)\right] dt, y_n(0) = 0$$

$$p^n: z_n(x) = \int_0^x (x-t)\left[-\varepsilon\left(\sum_{j=0}^{n-1} z_j\, z_{n-j-1}\dot{z}_{n-j-1} - \dot{z}_{n-1}\right) - z_{n-1} + k(z_{n-1} - y_{n-1})\right.$$
$$\left. + \mu\left(\sum_{j=0}^{n-1}\sum_{k=0}^{j} z_j\, z_{j-k} z_{n-j-1} - \sum_{j=0}^{n-1}\sum_{k=0}^{j} y_j\, y_{j-k} y_{n-j-1}\right)\right] dt, z_n(0) = 0\,, n \geq 2$$

The following solutions are obtained:

$$y_0(x) = \alpha,$$
$$z_0(x) = c\alpha + x\alpha\omega d$$
$$y_1(x) = \frac{1}{2}x^2(-\alpha + k\alpha - c\alpha k + \alpha^3\mu - c^3\alpha^3\mu)$$
$$+ \frac{1}{2}x^3\left(-\frac{1}{3}k\alpha d\omega - c^2 d\alpha^3\mu\omega - \frac{1}{2}cd^2\alpha^3\mu\omega^2 - \frac{1}{10}d^3\alpha^3\mu\omega^3\right)$$
$$z_1(x) = \frac{x^2}{2}(-c\alpha - k\alpha + ck\alpha - c^3\mu + c^3\alpha^3\mu + \alpha\varepsilon\omega - c^2 d\alpha^3\varepsilon\omega)$$
$$+ \frac{x^3}{2}\left(-\frac{1}{3}d\alpha\mu + \frac{1}{3}dk\alpha\omega + c^2 d\alpha^3\mu\omega - \frac{2}{3}cd^2\alpha^3\varepsilon\omega^2\right) + \frac{1}{4}x^4\left(cd^2\alpha^3\mu\omega^2 - \frac{1}{3}d^3\alpha^3\varepsilon\omega^3\right)$$
$$+ \frac{1}{20}x^5 d^3\alpha^3\mu\omega^3.$$

⋮

By means of the enclose Mathematica, in the identical approach one can get hold of the rest of the works. But in this folder, 3 systems of terms were worked out and the asymptotic solution is given by:

$$y(x,\alpha,\mu,\varepsilon,\omega,c,d,k) = y_0(x,\alpha,\mu,\varepsilon,\omega,c,d,k) + y_1(x,\alpha,\mu,\varepsilon,\omega,c,d,k) + y_2(x,\alpha,\mu,\varepsilon,\omega,c,d,k) + \cdots$$
$$z(x,\alpha,\mu,\varepsilon,\omega,c,d,k) = z_0(x,\alpha,\mu,\varepsilon,\omega,c,d,k) + z_1(x,\alpha,\mu,\varepsilon,\omega,c,d,k) + z_2(x,\alpha,\mu,\varepsilon,\omega,c,d,k) + \cdots$$

The following figures 1, 2, and 3 show the behaviour of the couple solutions as function of x first for fixed set of parameters and as a function of (x, ε).

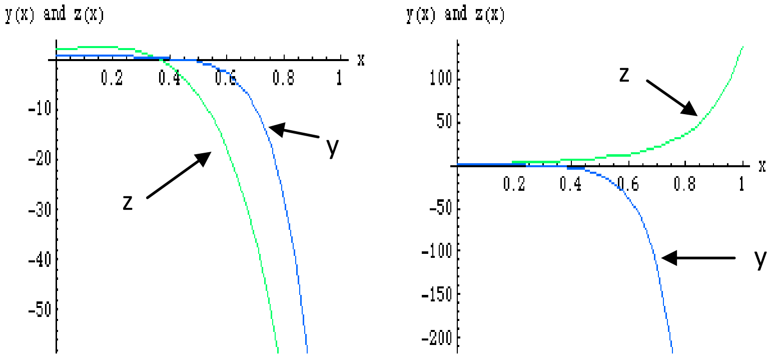

Fig. 1: Approximated coupled solutions, for (a) $\varepsilon = 1$ and (b) $\varepsilon = 0$

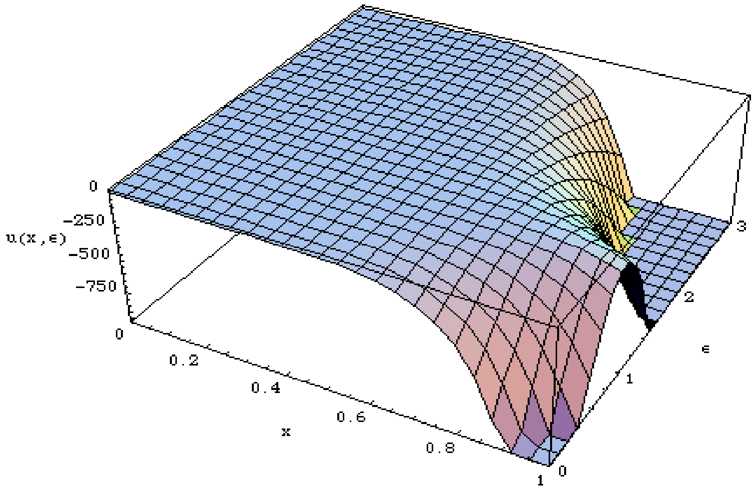

Fig. 2: y as function of x and ε

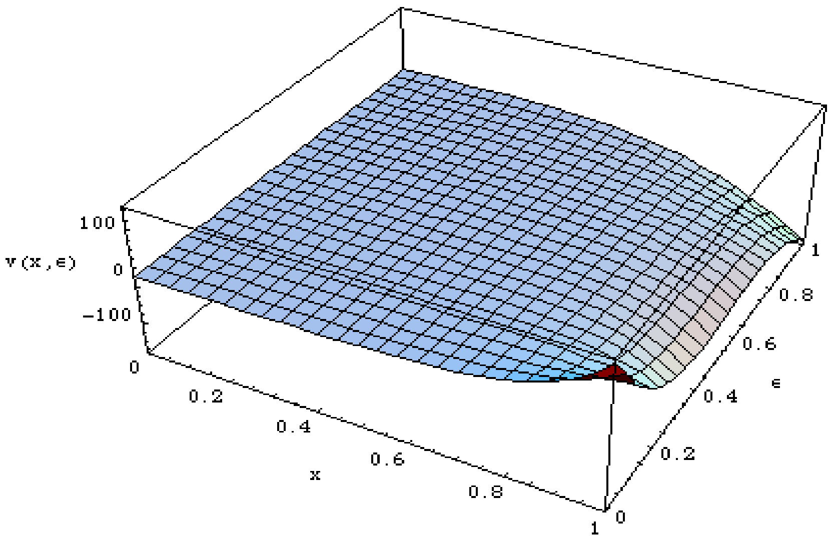

Fig. 3: y as function of x and ε

Problem 2: Consider the following model of a classical van der Pol oscillator coupled gyroscopically to a linear oscillator:

$$\begin{cases} \ddot{y}(t) + \varepsilon(y^2 - 1)\dot{y}(t) + y(t) + h\ddot{x}(t) = E\cos[nt] \\ \ddot{x}(t) + \lambda\dot{x}(t) + x(t) - dy(t) = 0. \end{cases}$$

Following the HDM steps, we obtained the following integral equations

$$p^0: y_0(t) = T(t), y_0(0) = B$$

$$p^0: x_0(t) = T1(t), y_0(0) = A$$

$$p^1: y_1(t) = -\int_0^t (t-\tau)\left(\varepsilon(y_0^2 - 1)\dot{y}_0 + y_0 + h\ddot{x}_0 - E\cos(n\tau)\right)d\tau, y_1(0) = B_1$$

$$p^1:\; x_1(t) = -\int_0^t (t-\tau)(\lambda\dot{x}_0 + x_0 - dy_0)d\tau, \qquad x_1(0) = A_1 \vdots$$

$$p^n: y_n(t) = -\int_0^t (t-\tau)\left(\varepsilon\left(\sum_{j=0}^{n-1}\sum_{k=0}^{j} y_j y_{j-k}\dot{y}_{n-1-j} - \dot{y}_{n-1}\right) + y_{n-1} + h\ddot{x}_{n-1}\right)d\tau, y_n(0) = B_n, n \geq 2$$

$$p^n: x_n(t) = -\int_0^t (t-\tau)(\lambda\dot{x}_{n-1} + x_{n-1} - dy_{n-1})d\tau,\; x_n(0) = A_n, n \geq 2$$

and the following solutions are obtained, by choosing the first term to be

$$y_0(t) = B\cos(t) + B_0\sin(t)$$

$$x_0(t) = A\cos(t) + A_0\sin(t)$$

$$\begin{aligned} y_1(t) = \frac{1}{8n^2}\Big(8E &+ n^2[-8(A + A_0 t - h(B + B_0 t)) - (A^2 - A_0(8 + A_0) + 4A(2 + A_0)t + 2(A^2 + A_0^2)t^2)\varepsilon] \\ &- 8E\cos(nt) \\ &+ n^2\big((A - A_0)(A + A_0)\varepsilon\cos(2t) + 8(2A_0 - B_0 h + A\varepsilon)\sin(t) \\ &+ 4\cos(t)(4A - 2B\,h - 2A_0\varepsilon + AA_0\varepsilon\sin(t))\big)\Big) \end{aligned}$$

$$x_1(t) = d(A + A_0 t) - B_0(t + \lambda) + B(-1 + t\lambda) + (2B - Ad + B_0\lambda)\cos(t) + (2B_0 - A_0 d - B\lambda)\sin(t).$$

As informed earlier , by means of the enclose Mathematica, in the identical approach one can get hold of the rest of the works. But in this folder, 3 systems of terms were worked out and the asymptotic solution is given by

$$y(t) = y_0(t) + y_1(t) + \cdots$$

$$x(t) = x_0(t) + x_1(t) + \cdots$$

The following figure 4 shows the behaviour of the couple solutions as function of t first for fixed set of parameters

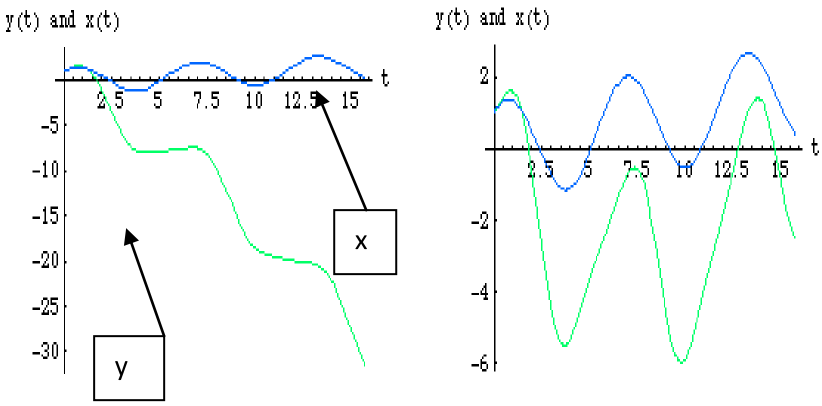

Fig. 4: Approximated coupled solutions, for (a) $\varepsilon = 0.1$ and (b) $\varepsilon = -0.1$

From the above figure we conclude that the coupled solution dependent on the value of ε, the shape of the coupled solutions and the sign change as the parameter ε changes.

Problem 3: Let us consider the periodic solutions of the two coupled van der Pol equations: In the chaotic state, the main feature is the high-sensitivity to initial conditions. This is the result of the combined effects of the cubic and intrinsic nonlinearities, and of the extrinsic periodic drive [18]. Consequently, a very small difference in the initial conditions will lead to different time histories or orbits. If two systems are launched with two initial conditions, they will circulate on different degenerated chaotic orbits. The goal of the synchronization in this case is to call one of the systems (slave) from its degenerated chaotic orbit to that of the other system (master) [18]. For this aim, the master system is described by the component while the slave system has the corresponding component y. The enslavement is carried out by coupling the slave to the master through the following scheme [18]

$$\begin{cases} \ddot{x}(t) + x(t) - \mu\left(1 - x^2(t)\right)\dot{x}(t) + \alpha x^3(t) = E_0 cos(t) \\ \ddot{y}(t) + (1+K)y(t) - \mu\left(1 - y^2(t)\right)\dot{y}(t) + \alpha y^3(t) - Kx(t) = E_0 cos(\varepsilon t). \end{cases}$$

The quantities μ and α are two positive coefficients, E_0 and ε are respectively the amplitude and frequency of the external excitation, where K is the feedback coupling coefficient.
In the view of homotopy decomposition method, the following integral equation are obtained

$$p^0 : x_0(t) = 1$$
$$p^0 : y_0(t) = 1$$
$$p^1 : x_1(t) = -\int_0^t (t-\tau)(-x_0 + \mu(1 - x_0^2)x_0' - \alpha x_0^3 + E_0 \cos(\tau))\, d\tau,$$
$$p^1 : y_1(t) = -\int_0^t (t-\tau)(-(1+K)y_0 + \mu(1 - y_0^2)y_0' - \alpha y_0^3 + E_0 \cos(\tau\varepsilon) + Kx_0)\, d\tau$$
$$\vdots$$
$$p^n : y_n(t) = -\int_0^t (t-\tau)\left(-(1+K)y_{n-1} + \mu\left(y_n' - \sum_{j=0}^{n-1}\sum_{k=0}^{j} y_j y_{j-k} y'_{n-j-1}\right) - \alpha \sum_{j=0}^{n-1}\sum_{k=0}^{j} y_j y_{j-k} y_{n-j-1} + Kx_{n-1}\right) d\tau$$
$$p^n : x_n(t) = -\int_0^t (t-\tau)\left(-x_{n-1} + \mu\left(x_{n-1}' - \sum_{j=0}^{n-1}\sum_{k=0}^{j} x_j x_{j-k} x'_{n-j-1}\right) - \alpha \sum_{j=0}^{n-1}\sum_{k=0}^{j} x_j x_{j-k} x_{n-j-1}\right) d\tau.$$

The following solutions are obtained,

$$x_0(t) = 1$$
$$y_0(t) = 1$$
$$x_1(t) = -E_0 + \frac{t^2}{2} + \frac{\alpha t^2}{2} + E_0 \cos(t)$$
$$y_1(t) = \frac{t^2}{2} - \frac{t^2\alpha}{2} - \frac{E_0}{\varepsilon^2} + \frac{E_0 cos(t\varepsilon)}{\varepsilon^2}$$
$$\begin{aligned} x_2(t) = E_0(-1 - \alpha(-4 - 2E_0 + t^2 + (-6 + t^2)\alpha) \cos(t) \\ + \frac{1}{120}\left(45E_0^2(-5 + 2t^2)\alpha - 10E_0\left(-12 + 48\alpha + 72\alpha^2 + t^4\alpha(1+\alpha) + 6t^2(1+2\alpha)\right)\right) \\ + t^4(1+\alpha)\left(5 + \alpha\left(10 + t^2(1+\alpha)\right)\right) - 15E_0^2\alpha\cos(2t) + 4E_0 t\alpha(1+\alpha)\sin(t) \end{aligned}$$

$$y_2(t) = \frac{1}{120\varepsilon^6}\Big(-45E_0\alpha(5E_0 + 16(1+\alpha)) + 30E_0(4 + 4K + (8 + 3E_0t^2)\alpha)\varepsilon^2$$
$$- 10E_0t^2\left(6 + 6K + \alpha(12 + t^2(1+\alpha))\right)\varepsilon^4$$
$$+ \left(60E_0(-2 + t^2) + t^4(1+\alpha)\left(5 + \alpha(10 + t^2(1+\alpha))\right)\right)\varepsilon^6$$
$$+ 15E_0\Big(8K\varepsilon^6\cos(t) - 8\left(-2\alpha(3 + E_0 + 3\alpha) + \left(1 + K + \alpha(2 + t^2(1+\alpha))\right)\varepsilon^2\right)\cos(t\varepsilon)$$
$$- E_0\alpha\cos(2t\varepsilon) + 32t\alpha(1+\alpha)\varepsilon\sin(t\varepsilon)\Big)\Big)$$

⋮

Using the package Mathematica, in the same manner one can obtain the rest of the components. But in this case, 3 systems of terms were computed and the asymptotic solution is given by

$y(t) = y_0(t) + y_1(t) + y_2(t) + \cdots$

$x(t) = x_0(t) + x_1(t) + x_2(t) + \cdots$

Now if the parameters $K = \alpha = E_0 = 0$, then the approximated coupled solution is given as:

$$x(t) = 1 + \frac{t^2}{2} + \frac{t^4}{24} + \cdots = \sum_{n=0}^{\infty} \frac{t^{2n}}{(2n)!}$$

$$y(t) = 1 + \frac{t^2}{2} + \frac{t^4}{24} + \cdots = \sum_{n=0}^{\infty} \frac{t^{2n}}{(2n)!}$$

This is the exact solution of the couple for this case.

The following figures 5, 6 and 7 show the behaviour of the couple solutions as functions of t first for fixed set of parameters

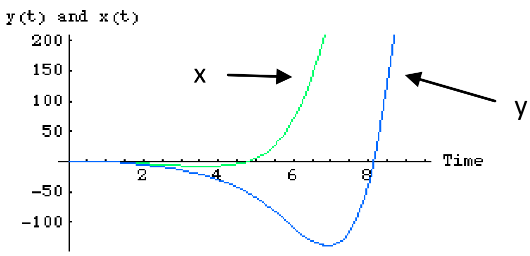

Fig. 5: Coupled solution

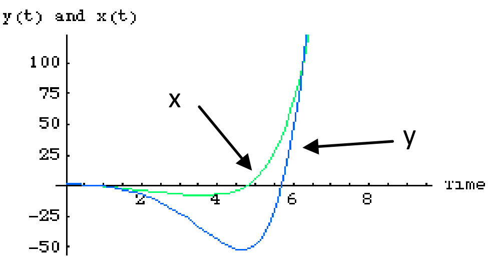

Fig. 6: Coupled solution

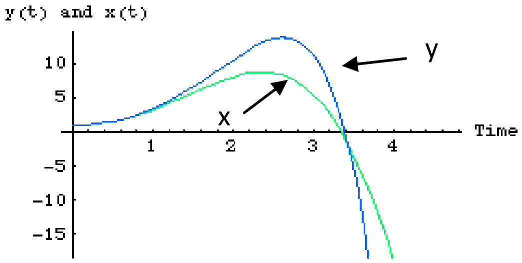

Fig. 7: Coupled solution

5. Conclusion

Numerical method for solving ODEs and PDEs can be viewed as accompaniment to get closer to exact solutions. Therefore in this section we present the advantages offered by HDM. The homotopy decomposition method is chosen to solve this nonlinear problem because of the following advantages the method has over the existing methods [19-20].

The purpose of this paper was to extend the homotopy decomposition method to coupled nonlinear van der Pol type's equation. The reliability of HDM and the reduction in computations give HDM a wider applicability. In addition, the calculations involved in HDM are very simple and straightforward [22]. Numerical solutions are given, and some properties exhibit physical reasonable dependence on the parameter values. Comparing the methodology HDM to PEM, HDM, ADM, VIM and HAM have the advantages. In contract to the PEM we do not need to expand all the parameters which some time leads to heavy nonlinear or linear ODEs or PDEs that are very difficult to solve. Unlike the ADM, the HDM is free from the need to use Adomian polynomials [14]. In this method we do not need the Lagrange multiplier, correction functional, stationary conditions, or calculating integrals, which eliminate the complications that exist in the VIM [23-24]. In contrast to the HAM, this method is not required to solve the functional equations in iteration each the efficiency of HAM is very much depended on choosing auxiliary parameter [21,24]. In contrast of HPM this method does not require, any deformation of a difficult problem to the easier one and also the equations obtained after comparing the terms of same power are very easier to solve [20,24]. We can easily conclude that the Homotopy decomposition method is an efficient tool to solve approximate solution of nonlinear differential equations [14].

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