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MODELLING AND SIMULATION OF THE BEHAVIOUR OF THE HYDRAULIC TURBINE

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Abstract

In this paper, a hydraulic turbine unit transfer function is represented, considering acceleration of the water column, velocity of the water and mechanical power.
Using Matlab/Simulink software facilities, have been simulated the behaviour of the mechanical power, depending of the gate opening, with water starting time parameter affected by uncertainties. Step responses analyse are simulated in order to evaluate the performances with load affected by uncertainties and with different control algorithms. This paper can be used in power engineering and control systems fields as a guideline in order to modelling, simulate, analyse and tune the controller for hydraulic turbine unit.

Key words: hydraulic turbine, mathematical modelling, hydro-electric power plant, control system

1. Introduction

The details about hydro-electric power plant structures, the basics about modelling and simulation of the hydro-electric power plant and the control techniques, considering different models and tuning algorithms, are presented in [1], [3], [4], [7]-[11], [13], [16], [17]. The main components of the hydro-electric plant are presented in fig. 1, where the prime source of electrical energy is the kinetic energy of water. Water is drawn to the turbine through the water column which includes intake structure, a penstock etc. The hydraulic turbine converts the kinetic energy of water into mechanical energy that is converted to electrical energy by generator [12].

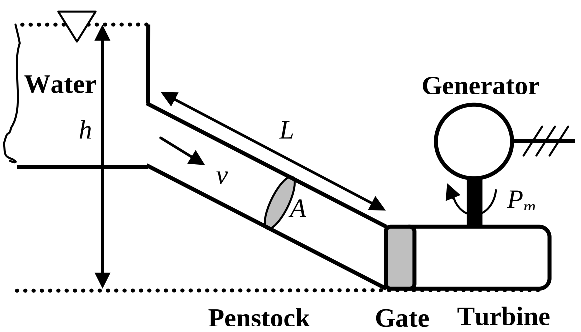

Fig. 1: Hydro-electric plant

The dynamic behaviour of the hydro-electric plant depends, mainly, by: transient of water flow in water column; velocity (v) of water in the pipe with length (L) and area (A); turbine mechanical power (P_m) [9].

The performance of a hydraulic turbine unit is influenced by the effects of water inertia and elasticity of structures, which gives the water hammer effect that influence the performances of the control system.

2. Mathematical modelling of hydraulic turbine unit

Several mathematical models, with details, for hydraulic turbine unit and the water column are presented in [1], [3]-[7], [9]-[11], [15]. Based on the complexity of the equations these models are detailed in: simplified linear, non-ideal linear, non-ideal elastic linear and nonlinear models.

This work presents the hydraulic turbine unit and water column considering the following hypothesis: the water is incompressible; the pipe is inelastic; the velocity of the water depends on the gate opening; the turbine mechanical power is proportional to the water flow.

According to Newton's law of motion, the dynamic equation for the *acceleration of water column* is [9]-[11]:

$$m \cdot \Delta a(t) + A \cdot \Delta p(t) = 0 \tag{1}$$

where: $m = \rho \cdot L \cdot A$ is the mass of water in the pipe; ρ – density; $\Delta a(t) = \frac{d}{dt}\Delta v(t)$ – infinitesimal change in acceleration (velocity); $\Delta p(t) = \rho \cdot g \cdot \Delta h(t)$ –

infinitesimal change in pressure at turbine gate; g – gravitational acceleration; $\Delta h(t)$ – infinitesimal change in high; t – time.

In order to normalize to the steady-state values, both sides of the relationship (1) are divided by $g \cdot h_o \cdot v_o$ (subscript "o" indicates steady-state values) and finally results [9]:

$$T_{ws} \frac{d}{dt} \Delta \bar{v}(t) = -\Delta \bar{h}(t) \quad (2)$$

where: $\Delta \bar{v}(t) = \frac{\Delta v(t)}{v_o}$ and $\Delta \bar{h}(t) = \frac{\Delta h(t)}{h_o}$ are the normalized values; $T_{ws} = \frac{L v_o}{g h_o}$ – the water starting time, which varies with load and represents the time required for h_o to accelerate the water in the penstock to the velocity v_o.

Typically, T_{ws} ranges between (0.5…6.0) sec. [1], [3], [9]-[12].

The steady-state and the dynamic relationships of the *velocity of the water* in the penstock are [9]-[11]:

$$v_o = k_v \cdot z_o \cdot \sqrt{h_o} \quad (3)$$

$$v(t) = k_v \cdot z(t) \cdot \sqrt{h(t)} \quad (4)$$

where: v is water velocity; z – gate position; k_v – proportionality constant.

For infinitesimal displacements around an operating point, after partial derivation and dividing by steady-state relationship (2) results:

$$\Delta v(t) = \frac{\partial v}{\partial h} \Delta h(t) + \frac{\partial v}{\partial z} \Delta z(t) \quad (5)$$

$$\Delta \bar{v}(t) = \frac{1}{2} \Delta \bar{h}(t) + \Delta \bar{z}(t) \quad (6)$$

where: $\Delta \bar{v}(t)$, $\Delta \bar{h}(t)$, $\Delta \bar{z}(t) = \frac{\Delta z(t)}{z_o}$ are the normalized values to steady-state values.

The steady-state and the dynamic relationships of the *turbine mechanical power* are [9]-[11]:

$$P_{mo} = k_p \cdot h_o \cdot v_o \quad (7)$$

$$P_m(t) = k_p \cdot h(t) \cdot v(t) \quad (8)$$

where k_p is a proportionality constant.

Considering infinitesimal displacements results:

$$\Delta P_m(t) = \frac{\partial P_m}{\partial h} \Delta h(t) + \frac{\partial P_m}{\partial v} \Delta v(t) \quad (9)$$

respectively,

$$\Delta \overline{P_m}(t) = \Delta \bar{h}(t) + \Delta \bar{v}(t) \quad (10)$$

where: $\Delta \overline{P_m}(t) = \frac{\Delta P_m(t)}{P_{mo}}$, $\Delta \bar{h}(t)$, $\Delta \bar{v}(t)$ are the normalized values to steady-state values.

Based on relationships (6) and (10) results:

$$\Delta \bar{h}(t) = 2\Delta \bar{v}(t) - 2\Delta \bar{z}(t) \quad (11)$$

$$\Delta \overline{P_m}(t) = 3\Delta \bar{v}(t) - 2\Delta \bar{z}(t) \quad (12)$$

Replacing equations (11) and (12) in equation (2) results the relationship between variation of mechanical turbine power and variation of gate position as:

$$0.5 \cdot T_{ws} \frac{d}{dt} \Delta \overline{P_m}(t) + \Delta \overline{P_m}(t) = \\ = -T_{ws} \frac{d}{dt} \Delta \bar{z}(t) + \Delta \bar{z}(t) \quad (13)$$

After *Laplace Transform*, the transfer function of an *ideal lossless hydraulic turbine* is:

$$H_{HT}(s) = \frac{\Delta \overline{P_m}(s)}{\Delta \bar{Z}(s)} = \frac{-T_{ws}s + 1}{0.5 \cdot T_{ws}s + 1} \quad (14)$$

According to [9]-[11], the transfer function for the non-ideal hydraulic turbine may be obtained by considering perturbed values for water velocity and turbine power.

3. Analysis of the transfer function of hydraulic turbine unit

In order to understand the basic characteristics of the hydraulic turbine is useful to study the behaviour of the transfer function (14) using linear analysis techniques as root locus, time and frequency responses. Also, this small - signal model is used to study the control tuning algorithms, but it is inadequate for large variations in power or frequency [9], [11]. The roots locus of the transfer function (14) shows zeros in the right half of the complex plane, so the system is a non-minimum phase one (fig. 2) [14].

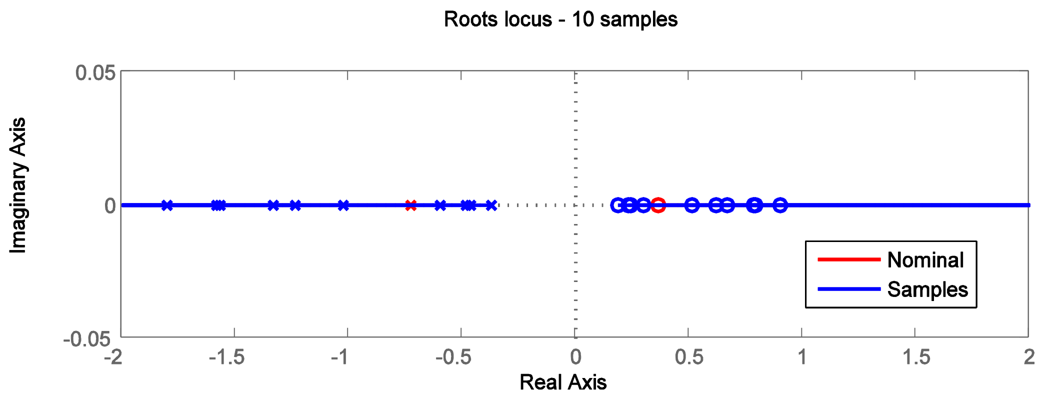

Fig. 2: Roots locus for of an ideal turbine with: $T_{ws} = [0.5...6]$ sec. and $T_{ws} = 2.75$ sec. as nominal value

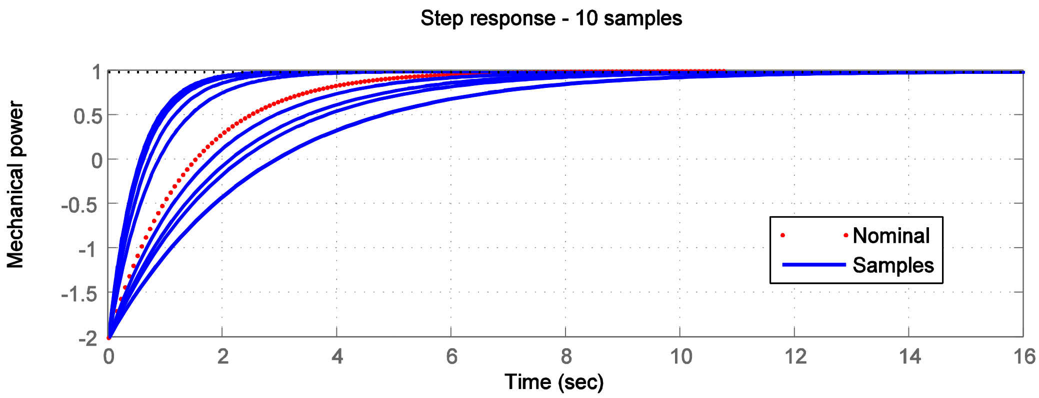

Fig. 3: The responses of an ideal turbine with $T_{WS} = [0.5...6]$ sec. and $T_{WS} = 2.75$ sec. as nominal value

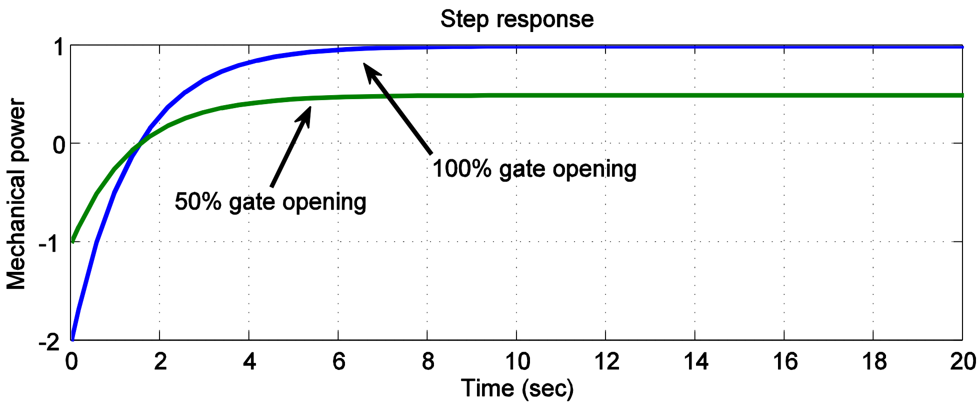

Fig. 4: The responses of an ideal turbine with $T_{WS} = 2.75$ sec., to different gate opening

The characteristics of the transfer function (14) may be illustrated by considering the responses to a step change in gate position, to different values for T_{WS} (fig. 3) and two different gate opening (fig.4).

4. Simulation of the hydraulic turbine unit control system

Figure 5 present the basic components of the hydro-electric power plant and control system, in which, the automatic control system refers to power and frequency control [9], [11], [12]. The basics for hydraulic turbine controller settings are presented in [1], [9]-[13], [16], [17], considering: stable operation during system-islanding conditions or isolated operation; acceptable speed of response for loading and unloading under normal synchronous operation.

The basic function of a controller is to control speed and/or load. The inside loop involves speed/load feedback, than speed error, which gives information about the gate position. The simplified block diagram of the speed control of the hydraulic turbine unit, supplying an isolated load with $H_L(s) = 1/(T_l s)$ is presented in fig. 6. The speed controller is provided with a droop characteristic. Typically, the steady-state droop is about 5% [2], [4] [9]-[11], [16].

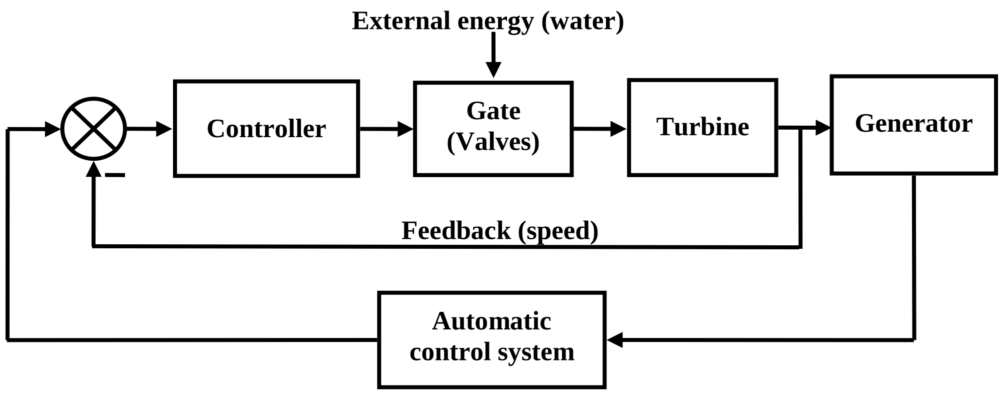

Fig. 5: Block diagram of hydro-electric power plant and control system

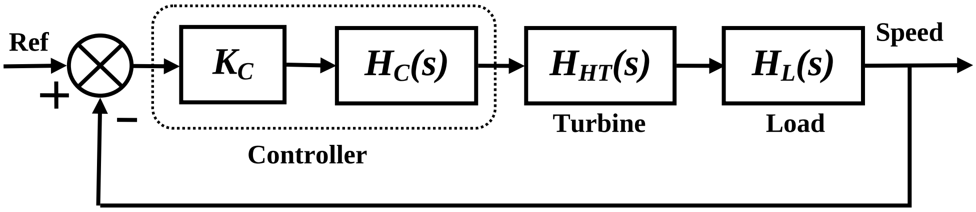

Fig. 6: The block diagram of the speed control

The controller $H_c(s)$ includes an integrative component $H_i(s)$, with time constant (T_i) and a transient droop compensation $H_{dc}(s)$:

$$H_c(s) = H_i(s) \cdot H_{dc}(s) \tag{15}$$

with:

$$H_i(s) = \frac{1}{T_i s + 1} \tag{16}$$

$$H_{dc}(s) = \frac{T_r s + 1}{T_{dc} s + 1} \tag{17}$$

$$T_{dc} = K_c T_d T_r \tag{18}$$

According to [9], [11], for stable operation under islanding conditions, the proper choice for temporary droop T_d and reset time T_r results from relationships:

$$T_d = \frac{T_{ws}}{T_l}\left[2.3 - 0.15 \cdot (T_{ws} - 1.0)\right] \tag{19}$$

$$T_r = T_{ws} \cdot \left[5.0 - 0.5 \cdot (T_{ws} - 1.0)\right] \tag{20}$$

Some controllers are provided with proportional-integral-derivative action (PID).

General rules for tuning a PID controller are presented in [1], [9]-[11], [16]. These PID parameters are tuned using linear control techniques and require linear models for hydraulic turbine and water column. The derivative action is beneficial for isolated operation. However, the use of a high derivative gain leads to oscillations and possibly instability.

Based on previous consideration, with LTI Matlab techniques, is possible to determine the lowest and the highest values of the proportional tuning parameter K_c (with $H_c(s)=1$) for which the response is stable and varies from damped to 5% overshoot (fig. 7).

Figure 8 present the close loop response, considering T_{ws} and T_l affected by uncertainties, expressed as a percentage from nominal value [14].

Figure 9 present the closed-loop response with the controller $H_c(s)$, determined with relationships (15)…(20) and considering T_{ws} and T_l affected by uncertainties.

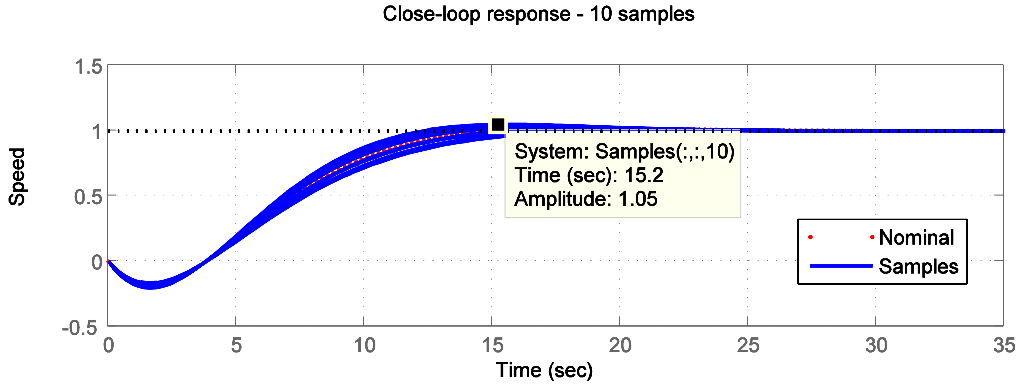

Fig. 7: The closed-loop response with: $K_c = [1.136...1.388]$, $T_{ws} = 2.75$ sec., $T_l = 10$ sec.

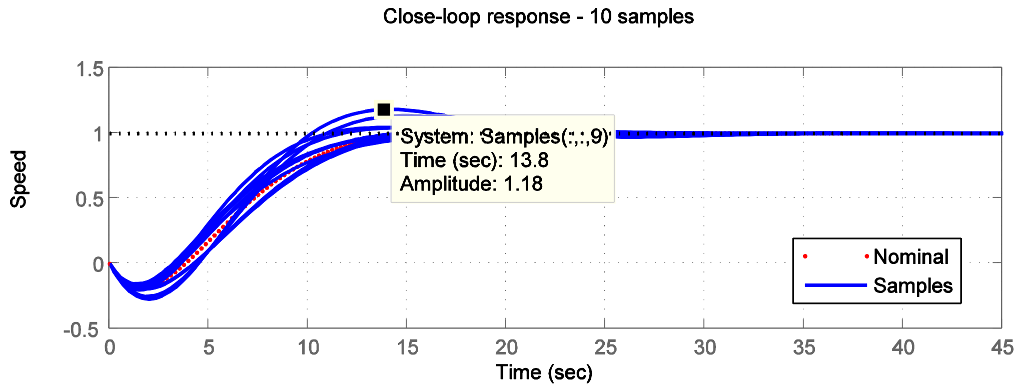

Fig. 8: The closed-loop response with: $K_c = [1.136...1.388]$, $T_{ws} = 2.75 \pm 20\%$ sec., $T_l = 10 \pm 20\%$ sec.

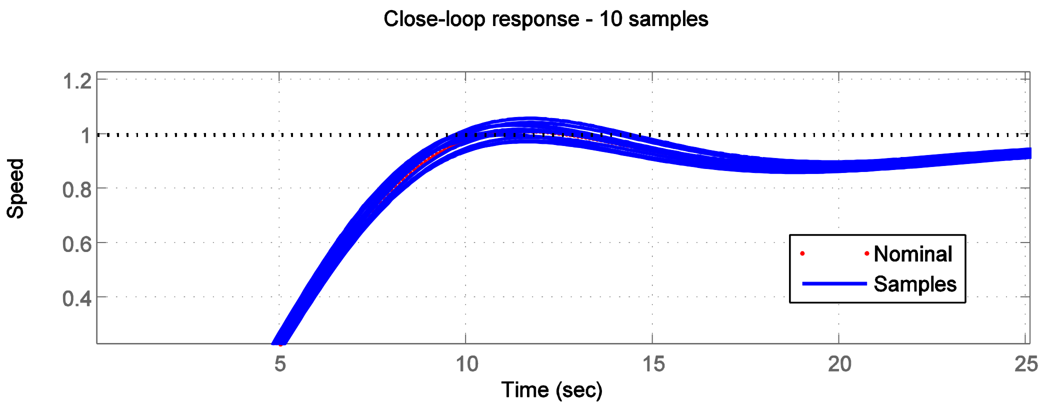

Fig. 9: The closed-loop response with: $K_C = 1.250$, $T_{WS} = 2.75 \pm 5\%$ sec., $T_l = 10 \pm 10\%$ sec.

5. Conclusion

The purpose of this paper is to present a simplified way to obtain the mathematical model which approximates the hydraulic turbine characteristics at low frequencies. The parameters of these models influence the dynamic simulation and need to be determined with accuracy.

The ideal transfer function of the lossless hydraulic turbine unit is used for performance studies with different types of controllers, tuned with well-known methods.

These models are not adequate for fast and stable response studies, so, the transfer function for the non-ideal hydraulic turbine should be obtained by considering perturbed values for water velocity and turbine power.

The illustrated responses show that, due to water inertia, hydraulic turbines have initial inverse characteristics of power to gate changes. The proper tuning of the controllers influences the stability and the performance of the connected power system.

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