

## MODELLING AND STUDYING TRANSIENT STATE OF SIMPLE SYSTEMS WITH LABVIEW

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### Abstract

*This paper presents virtual instruments to simulate the transient state for some simple and complex circuit. The relative short event between different steady states is the transient state which is followed by energy variation in the circuit. This energy variation produces impulses which propagate through the net and influence other devices. Because there are a lot of equipments operating in switching mode, it is important to study the transient state. To model and study the transient state equations for circuits were written. The article presents the mathematical base of the transient state and types of response of the transient state. The main part is to present the developed virtual instruments for electrical circuits to simulate and to study their transient state. The virtual instruments were developed in LabView. The virtual instruments allow setting different circuit configurations, different values for circuit elements and show the results in analytical and graphical way.*

**Keywords:** transient state, transient state model, virtual instrument, LabView simulation

### 1. Introduction

Electrical circuits are functioning in steady state of direct current or alternative current. By switching on or off instruments and devices, a state of short time appears, which is a transition state between two steady states. This relative short transition time is the transient state.

The transient state appears not only by on or off switching of the circuits, but also by any changing in a circuit. The transient state is a relatively short event of energy modification in a system caused by a sudden change in state. The length of the transient state is depending on the type of the circuit and the state which is going to reach.

The phenomena of the transient state affect the operating way of circuit elements or devices. The produced impulse and shock propagate and can damage instruments and equipments.

The transition from off state in on state or inverse is followed by charging and discharging of parasite capacitors. The magnetic field of induction coil is changing also; it is accumulating or dissipating energies. These phenomena are relative long and influences the circuit elements and equipments. A very simple example, by switching on an electrical motor, during the transient state, the starting current is six times larger than the nominal current.

Any changes in the system produce energy modification which appears as an oscillation of the quantities and output signals. In electrical engineering, oscillations are caused by a sudden change of the circuit structure and this change is studied through transient response of the circuit or the

system. The transient response gives the variation of the current through the induction coils and the variation of the voltage through the capacitors at a given input.

Different mechanical or electrical systems are studied at impulse and step input. These are specific inputs to analyze the type or the behavior of the system. The step input and inverse input step can be alike with on/off switching. The transient response is not necessarily bound to on/off switching but to any event that affects the state of the system. [3],[4]

The length of the transient state is theoretically infinite, but practically it can be considered that it influences the functioning of the system in a shorter period, which equals 4-5 times the circuit constant.

Reaching the new steady state depends on the system structure and on the parasite elements. The transient response can be aperiodic or oscillatory. In both cases the transient response is damped due to the loss through Joule-Lenz effect.

In electrical engineering is often used switching power supply, equipments which operates in switching mode, induction motors with variable speed or other circuits which produce oscillations during the working period. Voltage regulators and surge protectors are used to prevent transients in electricity and to protect delicate instruments.

Electromagnetic compatibility treats with performance and resilience of the systems to transient interference. In this case transients are deliberately applied to electronic equipment to testing their behavior. There are induced fast transient oscillations

in the form of sine wave. International standards define the magnitude and methods used to apply transients.

## 2. Mathematical base of transient state

The transient state can be modeled as a damped harmonic oscillator. The transient response of the system or circuit can be classified in three types which show how the steady state is reached. [1],[2]

The first is the undamped response with a damping ratio  $<1$ . In this case the steady state is reached through oscillations with decreasing amplitude. The more undamped the system is the oscillations takes longer.

The overdamped response is where the damping ratio is  $>1$  and the new steady state is reached without oscillations.

The critically damped response is between the undamped and overdamped response. It is similar to the overdamped response; the steady state is reached without oscillations but very fast. The damping ratio is  $=1$ . Figure 1 shows the undamped and the overdamped response type of a system. [4]

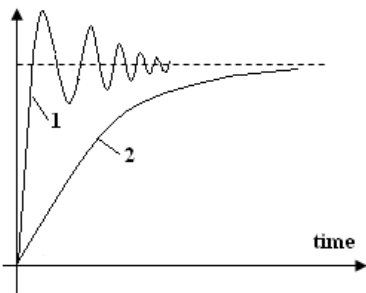


Fig.1. Undamped (1) and overdamped (2) response.

To analyze the system or circuit behavior the first step is to write the system equations. The general equation for a second order system is: [2]

$$m \frac{d^2y}{dt^2} + c \frac{dy}{dt} + ky = F \quad (1)$$

In this equation  $m$  is the mass,  $k$  is the main spring and  $c$  is the damping of the system,  $F$  represents the input force or signal.

For a second order system it is important to determine the dc gain, the damping ratio and the natural frequency. Equation (1) can be written:

$$\frac{d^2y}{dt^2} + 2\xi\omega_n \frac{dy}{dt} + \omega_n^2 y = G\omega_n^2 u(t) \quad (2)$$

In this equation  $\omega_n = \sqrt{k/m}$  is the natural frequency

and  $\xi = \frac{c}{2\sqrt{km}}$  damping ratio and  $G$  is the dc gain.

The solution of this differential equation has two parts the transient solution and the steady state solution which is reached after five time the circuit constant.

In electrical engineering the different equipments which operate in switching mode have

inductive and capacitive parts with nonlinear characteristics. The simplest second order differential equation for an electrical circuit is:

$$L \frac{d^2i(t)}{dt^2} + R \frac{di(t)}{dt} + \frac{1}{C} i(t) = u(t) \quad (3)$$

It is known that coils and capacitors have nonlinear characteristics which make the solutions more complicate.

An important step in solving differential equations is to determinate the constants which are depending from initial conditions; in electrical circuits from the energy stored in magnetic and electrical fields. The transient state depends on the variation of this energy.

There are two methods to solving differential equations written for electrical circuits. The first method is the direct integration of the equation, to determine the transient part and the steady state part separate and the final solution is the sum of these two parts. This method is used in case of circuits with relative simple structure. The second method used the Laplace transform. The solution is determinate as a simple fraction or sum of simple fractions and the original function will be an exponential form. The constants are calculated using the initial conditions. This second method is used in case of complex circuits. [9]

For simple circuits the equation of the system is written and a general solution is deduced which has the following form:

$$i(t) = i_p + (i_0 - i_{p0})e^{-\frac{t}{\tau}} \quad (4)$$

In this equation  $i_p$  is the steady state response,  $i_{p0}$  is the initial value of the steady state,  $i_0$  is the initial value of the current and  $\tau$  is the circuit constant.[5]

## 3. Transient response of simple circuits

The behavior of circuits and systems at transient state is studied through models. These circuit models can be simple or complex depending on the structure of the system. The simplest circuits are the RL, RC and RLC circuits. These circuits are studied at on/off switching. Figure 2 shows the RL circuit on/off switching. [5],[6],[8]

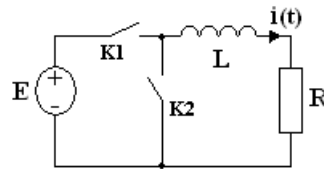


Fig.2. The on/off switching of the RL circuit.

The equation written for this circuit without initial condition is:

$$L \frac{di(t)}{dt} + Ri(t) = E \quad (5)$$

The solution of this equation, the variation of the current through the circuit at on switching and the variation of the voltage on the coil are:

$$\begin{aligned} i(t) &= \frac{E}{R} \left(1 - e^{-\frac{R}{L}t}\right) \\ u_L(t) &= L \frac{di}{dt} = E e^{-\frac{R}{L}t} \end{aligned} \quad (6)$$

Figure 3 shows these variations.

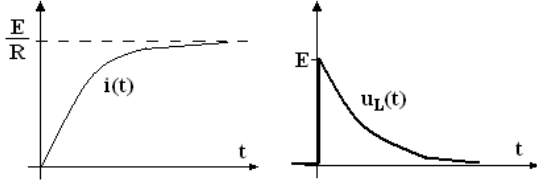


Fig.3. The current and voltage variation.

The damping ratio depends on circuit constant,  $\tau = \frac{L}{R}$ .

By off switching the circuit, there is initial condition (there is a magnetically energy stored in the coil), and the circuit equation is:

$$L \frac{di(t)}{dt} + Ri(t) = 0 \quad (6)$$

The equation will be solved in the same way, as the equation (5). To calculate the integration constants the initial current must be considered.

The equations for an RC circuit are:

$$\begin{aligned} Ri(t) + \frac{1}{C} \int i(t) dt &= u(t) \\ R \frac{dq}{dt} + \frac{1}{C} q &= u(t) \end{aligned} \quad (7)$$

In this case the solution will give the variation of the charge and from this will be determine the current and voltage variation. The length of the transient state depend on the circuit constant,  $\tau = RC$ .

Figure 4 shows the on/off switching of the RLC circuit.

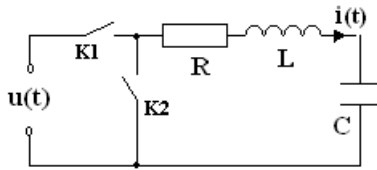


Fig.4. On/off switching of the RLC circuit.

The differential equation written for this circuit by on switching is:

$$L \frac{d^2 q(t)}{dt^2} + R \frac{dq(t)}{dt} + \frac{1}{C} q(t) = u(t) \quad (8)$$

The transient state solution depends on the value of the circuit elements. The general form of the transient state solution is:

$$q_t(t) = e^{-\delta t} (Ae^{\omega t} + Be^{-\omega t}) \quad (9)$$

In this equation A and B are constants depending on the initial conditions,  $\delta = \frac{R}{2L}$  is the damping ratio,  $\omega = \sqrt{\delta^2 - \omega_n^2}$  is the pseudo frequency of the circuit,  $\omega_n = \frac{1}{\sqrt{LC}}$  is the natural

frequency of the circuit. [5],[8],[9]

In accordance with the value of the circuit elements the solution of the differential equation can be of three types. If the damping ratio is  $\delta < 1$  the solution is oscillating with decreasing amplitude. In this case  $\delta < \omega_n \Rightarrow \frac{R}{2L} < \frac{1}{\sqrt{LC}}$ .

If the damping ratio is  $\delta = 1$  the solution is critical non periodic and  $\frac{R}{2L} = \frac{1}{\sqrt{LC}}$ .

If the damping ratio is  $\delta > 1$  the response is overdamped and the solution is non periodic. The relation between the circuit elements for the overdamped case is  $\frac{R}{2L} > \frac{1}{\sqrt{LC}}$ .

Connecting the RLC circuit to a continuous voltage supply the initial conditions are  $q_0=0, i_0=0$ . At the steady state  $q=CE$  and  $i=0$ .

The general solution for the transient state is:

$$q_t = e^{-\delta t} \left[ (q_0 - q_{p0}) \cos \omega t + \frac{\delta}{\omega} (q_0 - q_{p0}) \sin \omega t + \frac{1}{\omega} \sin \omega t \right] \quad (10)$$

From the charge variation can be determine the current variation using the relation  $i(t) = \frac{dq(t)}{dt}$ .

#### 4. Virtual instruments to model the transient state

Based on mathematical equations deduced for different electrical circuits virtual instruments was developed to follow the current, voltage and charge variation in the circuit by on/off switching. [6]

These virtual instruments have three sides. In the first side can be set the time range in which the transient state will be studied and the estimation step. Figure 5 shows this first side from the virtual instrument developed for studying the RL circuit. Here can be set the switching type of the circuit. [10]

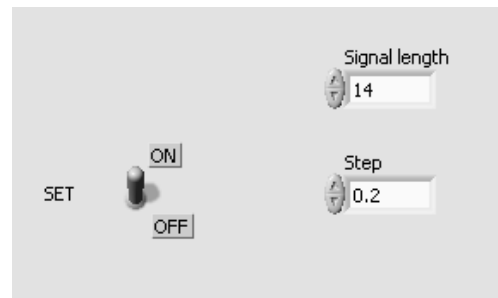


Fig.5. Time and step setting panel.

The second side is to insert the circuit elements value, the supply voltage value and the initial conditions if there are.

The results, steady state values and the variation manner of the electrical quantities are plotted in the third side of the front panel of the virtual instrument. In figure 6 is presented the transient state for the RL circuit.

In figure 7 is presented a part of the front panel of the virtual instrument developed for studying RC circuit. There are plotted the circuit constant, the

amplitude of the transient state and the steady state and the variation of the current, voltage and charge.

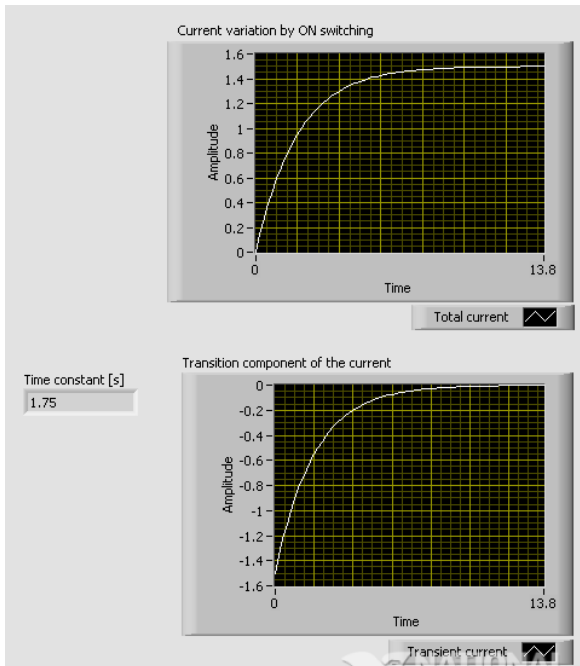


Fig.6. Current variation in the RL circuit.

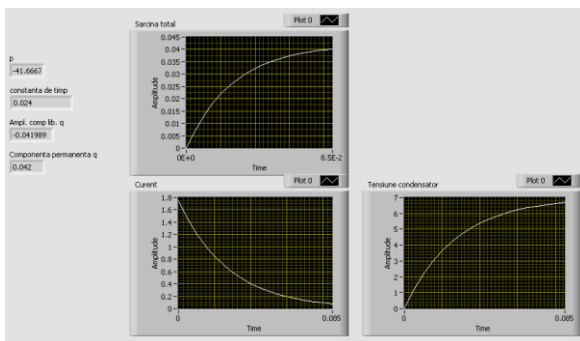


Fig.7. Current, voltage and charge variation in the RC circuit.

To simulate the RLC circuit a second order differential equation was used (relation 8). For this circuit is possible three response type depending on the circuit elements value. In figure 8 is presented the critical damped response.

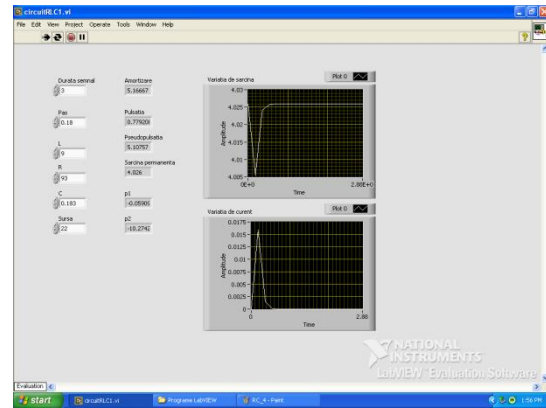


Fig.8. Critically non periodic response of the RLC circuit.

Figure 9 present the front panel of the virtual instrument for the RLC circuit with oscillating response.

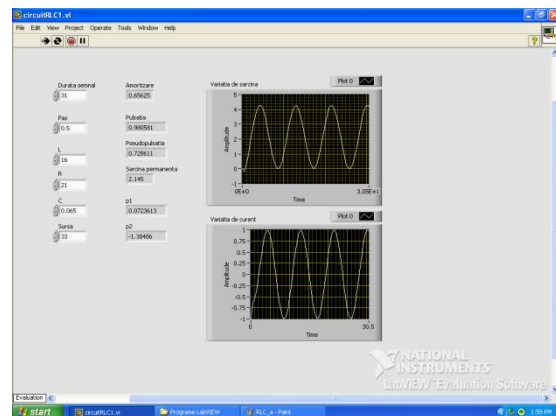


Fig.9. Periodical response of the RLC circuit.

To each virtual instrument front panel belongs a bloc diagram, the graphical soft. In figure 10 is presented a part of the bloc diagram of the virtual instrument developed for RLC circuit. [10]

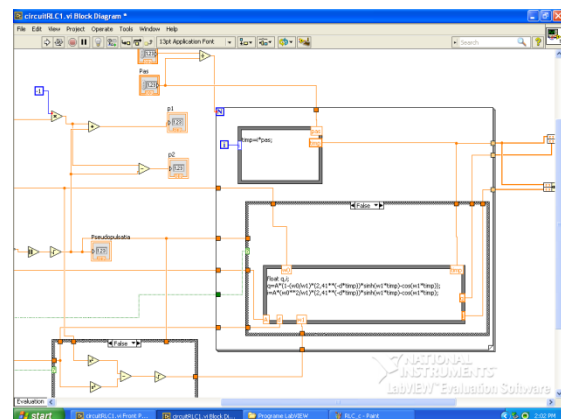


Fig.10. Bloc diagram of the RLC circuit virtual instrument.

## 5. Conclusions

The transient state appears by on/off switching and also by any changing in a circuit. This relative short event of modification of energy in a system affects the operating way of devices. The produced impulse and shock propagate and can damage instruments and equipments. The length of the transient state is depending on the type of the circuit and the reached steady state.

The mathematical base used by transient state study is similar to the mechanical system vibration. In both cases a second order differential equation can be written. There are similarity between mass, main spring, damping and inductivity, resistance and capacity.

Based on mathematical relations virtual instruments were developed for simple and complex electrical circuits using LabView.

Through these virtual instruments can be studied the transient state by any changing in circuits. It can be determined the length and type of the transient state. The impulses or oscillations can be studied, which appear at the modification in the circuit structure and the values of the circuit elements can be determined to reduce the negative effects.

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