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STUDYING TRANSIENT STATE USING STATE PARAMETERS

Katalin ÁGOSTON
"Petru Maior" University of Tîrgu-Mureș, Romania
Nicolae Iorga Street, no.1, 540088 Tîrgu-Mureș, Romania
katalin.agoston@ing.upm.ro

Abstract

This paper presents the state variables method in the analysis of the transient state. Using this method the behaviour of transformers at on/off switching is studied. The transient state is a short event caused by any changes in the system which is followed by an energy modification stored in the system. The energy is stored in capacitors and coils and any modification needs time and has reactions. These reactions can be high impulses of energy which propagate through the network and influence or even destroy other circuit elements or devices. The transient state of an RLC circuit and of a transformer is studied. Differential equations are used and the nonlinearity of the coil is highlighted. Differential equations and the equivalent circuit are used to deduce the state parameter model for the transformer. Equations and equivalent circuit for a real transformer are deduced with and without load at the output. The behaviour of the transformer was studied while functioning in an open circuit and with different loads at the output. To study the output parameter variations state space model simulation was used.

Keywords: transient state model, state parameters, transformer, Simulink model

1. Introduction

The transient state is a short event in a system caused by a sudden change of the functioning state. The transient state can be also characterized through a relatively short event of energy modification in a system caused by a sudden change in state. By switching on/off the circuit elements or devices, a short transition state appears between the two steady states. This relative short transition time theoretically is infinite, but practically it is 4-5 times longer than the circuit constant.

The transient state appears not only by on/off switching of the circuits, but also by any changing in the circuit. The length of the transient state is depending on the type of the circuit and the state which it is going to reach.

The produced impulse and shock propagate through the net and can damage instruments and equipments. The phenomena of the transient state affect the operating way of the circuit elements or devices also. [5]

The characteristics of the transient state depend on the charging and discharging of parasite capacitors, and the change of the magnetic field of the induction coil. These changes are relative long and influence the circuit elements and equipments. For example, coupling an electrical motor, the starting current is six times larger than the nominal current, and this shock propagates in the electrical network a long distance. This shock appears at other equipments like overvoltage.

Any change and energy modification in the system appears as an oscillation. Mathematically, it can be modeled as a damped harmonic oscillator. In electrical engineering, oscillations are caused by a sudden change of the circuit structure and this change is studied through transient response of the system. The transient response gives the variation of the output signal at a given variation of input.

There are many types of equipment which operating mode is followed by short or large transient state. For example there are switching power supply, induction motors with variable speed, equipments which operate in switching mode, or other circuits which produce voltage or current oscillations during the working period.

Voltage regulators and filters are used to prevent transients in electricity and to protect instruments and other equipments.

Different mechanical or electrical systems are studied applying impulse and/or step input. These are specific inputs used to analyze the behavior of the system. The step input and inverse step input can be like on/off switching. The transient response is not necessarily bound to on/off switching but to any event that affects the state of the system. [3],[4]

The graphic form which shows how the new steady state is reached depends on the system structure and on the parasite elements. The transient response or natural response of a system can be of three types. At non-periodic or overdamped response, the new steady state is reached without oscillations in a longer time. At critically damped response the steady state value is reached without oscillations but in the fastest way, without overdamping. The damping ratio in this case is equal to one. The third type of transient response is the underdamped response; the steady state is reached with oscillations, but the magnitude of the oscillations decrease. In all the cases the transient response is damped due to the loss through Joule-Lenz effect.

To check the behavior of systems for transient cases produced by other systems, there are deliberately applied transient oscillations to electronic equipment. Fast transient oscillations are induced in the form of sine wave. International standards define the magnitude and methods used to apply transients.

2. Bases of the transient states

The transient state can be compared with a damped harmonic oscillator. The transient response of a system or a circuit can be classified in three types which show how the new steady state is reached. [1], [3]

The first is the undamped response; in this case the steady state is reached through oscillations with decreasing amplitude. The damping ratio is lesser than one. The more undamped the system is the oscillations take longer.

The overdamped response is where the damping ratio is higher than one and the new steady state is reached without oscillations.

The critically damped response is between the undamped and overdamped response. It is similar to the overdamped response; the steady state is reached without oscillations but very fast. The damping ratio is equal to one. Figure 1 shows the undamped and the overdamped response type. [4], [6]

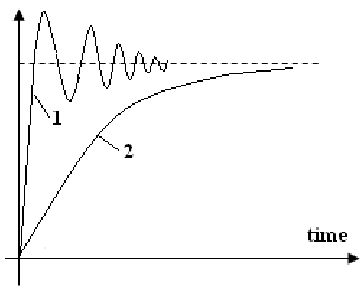

Fig. 1: Undamped (1) and overdamped (2) response

To analyze the system or circuit behavior the first step is to write the system equations. The general equation for a second order system is: [3]

$$m\frac{d^2y}{dt^2} + c\frac{dy}{dt} + ky = F \qquad (1)$$

In this equation m is the mass, k is the main spring and c is the damping of the system, F represents the input force or signal.

For a second order system it is important to determine the dc gain, the damping ratio and the natural frequency. Equation (1) can be written:

$$\frac{d^2y}{dt^2} + 2\xi\omega_n\frac{dy}{dt} + \omega_n^2 y = G\omega_n^2 u(t) \qquad (2)$$

In this equation $\omega_n = \sqrt{k/m}$ is the natural frequency and $\xi = \frac{c}{2\sqrt{km}}$ is the damping ratio, and G is the dc gain.

The solution of this differential equation has two parts: the transient solution and the steady state solution which is reached after longer or shorter time depending on system or circuit elements.

In electrical engineering the different equipments which operate in switching mode have inductive and capacitive parts with nonlinear characteristics. For each complex circuit can be deduced an equivalent circuit with linear or nonlinear coefficient through which the behavior of the circuit or system can be followed easier. For the simplest RLC electrical circuit can be written the equation:

$$u(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt$$
$$i(t) = \frac{dq(t)}{dt} \qquad (3)$$

Where *i(t)* is the current through the circuit and *q(t)* is the capacitors charges. The simplest RLC electrical circuit is described through a second order differential equation:

$$\frac{d^2q(t)}{dt^2} + \frac{R}{L}\frac{dq(t)}{dt} + \frac{1}{LC}q(t) = \frac{1}{L}u(t) \qquad (4)$$

The current variation is obtained in two steps. First the charge variation is deduced and from this the current variation is determined.

There are more methods of solving differential equations written for electrical circuits. These methods are grouped in two categories, analyzing methods in time domain and analyzing methods in frequencies domain.

The first and simplest method in time domain is the direct integration of the equation. The transient part and the steady state part are calculated separately and the final solution will be the sum of these. This method can be used for circuits with relative simple structure. [6] For RL circuits the general solution for the current has the following form:

$$i(t) = i_p + (i_0 - i_{p0})e^{\frac{t}{\tau}} \qquad (5)$$

In this equation i_p is the steady state response, i_{p0} is the initial value of the steady state, i_0 is the initial value of the current and τ is the circuit constant.[5] If these values are known, the variation of the current in transient state can be obtained easy, regardless of the form of the input signal u(t).

For RC circuit the general solution has the same form, but the charge variation is obtained. From this is deduced the current variation and the voltage variation through the capacitor.

It is known that coils and capacitors have nonlinear characteristics which make the solutions more complicate.

An important step in solving differential equations is to determinate the constants which are depending from initial conditions. The transient state depends on the variation of the energy stored in magnetic and electrical fields.

For complex circuits the Laplace transformation is used. Making these transformations simple equations can be written for the circuit. The solution is obtained as a simple fraction or sum of simple fractions. The original function is obtained with Heaviside theorem and will have an exponential form, or a sum of exponentials.

$$f(t) = \sum_{k=1}^{n} C_k e^{s_k t} \qquad (6)$$

The constants C_k are depending from initial conditions. [9]

3. Studying transient state using state parameters

The output of a system depends on the input and the state of the system. The relation between these is input u(t) → state x(t) → output y(t) and the system is described through the functions:

$$\dot{x} = f(x,u) \;\; and \;\; y = g(x,u)$$

For linear systems the functions *f* and *g* will be constant and the system is described through:

$$\dot{x} = Ax + Bu \;\; and \;\; y = Cx$$

If the output depends on the state of the system and on the input also, the system equation is:

$$\dot{x} = Ax + Bu \;\; and \;\; y = Cx + Du$$

The order of the systems and the number of state parameters depend on the number of energy accumulation possibilities. In electrical circuits energy is accumulated in magnetic fields of coils and in the electrical fields of capacitors.

Describing linear and nonlinear electrical circuits using state parameter models can be developed. Through these models the behaviour of circuits and systems at transient state can be studied.

The state parameters express the voltage on capacitors and the current through coils. [7], [9], [10]

The simplest RLC circuit is studied through a second order differential equation like equation (4). This second order differential equation can be written with two first order differential equations:

$$\frac{du_C}{dt} = \frac{1}{C} i$$
$$\frac{di(t)}{dt} = -\frac{R}{L} i - \frac{1}{L} u_C + \frac{1}{L} u(t) \qquad (7)$$

The state parameters are the voltage u_c and the current *i*. The input is *u(t)*. The equations (7) can be written in matrix form:

$$\frac{d}{dt}\begin{bmatrix} u_C \\ i \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix} \cdot \begin{bmatrix} u_C \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} \cdot u(t) \qquad (8)$$

Modeling and studying the system using this form of the system equation is easier. Figure 2 shows the Simulink model of the RLC circuit.

Applying various input signal *u(t)* the behavior of the system can be studied. Figure 3 present the variations of the current and the capacitor voltage if the input voltage has a sinusoidal form with low frequency.

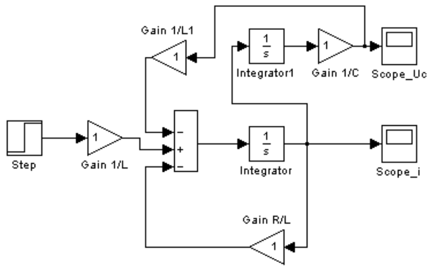

Fig. 2: The Simulink model of the RLC circuit

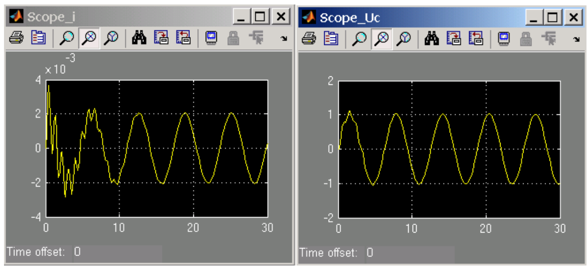

Fig. 3: The current and voltage variation for low frequency sinusoidal input

If the frequency is increasing the transient state will be more emphasized, like in figure 4. The higher value of the current during the transient state can be risky for the circuit elements.

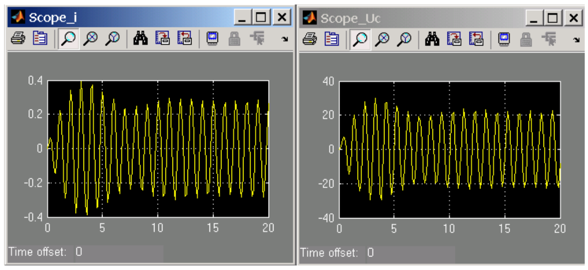

Fig. 4: The current and voltage variation for higher frequency sinusoidal input

4. Modeling the transformer with state parameters

There are many practical cases where the circuit elements do not have a constant value; they are function of time, current or voltage. These nonlinear circuit elements can cause overvoltage or non-sinusoidal current. The coil with magnetic core is an important circuit element which stores energy in magnetic field. At ON switching the absorbed current is higher than in steady sate to create the magnetic field. There is a phase difference between the voltage and current variation because the stored energy cannot vary immediately. This produces another abnormality. At ON switching the magnetic field rises from zero to the maximum value which leads to an overvoltage according to the induction law:

$$u_{e_\Gamma} = -\frac{d\phi_{S\Gamma}}{dt} \qquad (9)$$

The same phenomenon appears at OFF switching, but in opposite sense.

The magnetic core material has nonlinear magnetization B=f(H) characteristics, and has also a magnetic hysteresis loop which shows that magnetization and the magnetic flux depend not only from the magnetic field strength, but also from the previous magnetization of the core material. [2], [8]

Figure 5 shows the B=f(H) characteristic of a transformer core from ferrimagnetic material. This is a nonlinear characteristic which can be approximated with a 5 order polynomial equation. The same nonlinearity exists between current and voltage, because B=f(i) and H=f(u). The position of the operating point on the characteristic influences the form of the current through the circuit which contains this magnetic core.

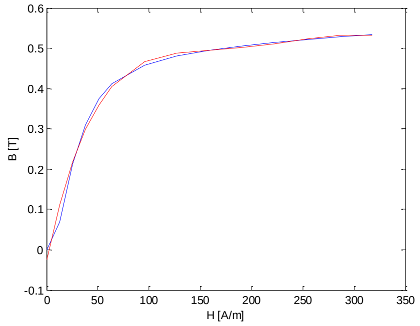

Fig. 5: The B=f(H)characteristic of ferrimagnetic material and the curve which approximate it

In some cases the position of the operating point is on the nonlinear part of the characteristic and the current through the circuit will be distorted.

In core material energy dissipation appears due to the magnetic hysteresis also, which is in the form of heat. The wasted energy is proportional with the area of the magnetic hysteresis loop. Hysteresis losses are always an important problem in coils and transformers where the current constantly changes the direction. [2]

The transformer uses magnetic cores from ferromagnetic or ferrimagnetic material, which operate nearly to the saturated part of the magnetization curve. The ON/OFF switching of these circuits cause complex transient state, where the type of the load is also important.

If no load is connected to the secondary winding, the transformer is functioning in open circuit and the transformer can be compared with RL circuit. The equation written for the RL circuit by on switching is:

$$Ri + L\frac{di}{dt} = Usin(\omega t + \alpha) \qquad (10)$$

Considering that the coil has N coiling and the magnetic flux through the coil is Φ, the inductivity is expressed with:

$$L = \frac{N\Phi}{i} \qquad (11)$$

The equation (10) becomes the following form:

$$Ri + N\frac{d\Phi}{dt} = Usin(\omega t + \alpha) \qquad (12)$$

Because Φ is a function of the current through the coil $\Phi=f(i)$ and this is unknown, the equation (12) cannot be solved. The variation of the flux will be sinusoidal after the ending of transient state, but the damping in the transient state is depending on R.

To study the flux variation in the transient state two cases will be considered. For the first time we consider that the flux has a linear variation, which is real at the beginning or at the linear part of the magnetization curve. Here can be used the relation $\Phi=Li$. In this case equation (12) can be written as:

$$R\frac{N\Phi}{L} + N\frac{d\Phi}{dt} = Usin(\omega t + \alpha) \qquad (13)$$

Reordering the equation it is obtained a similar function as for RL circuit. The matrix form of this equation is:

$$\frac{d}{dt}[\Phi] = \left[-\frac{R}{L}\right]\cdot[\Phi] + \left[\frac{1}{N}\right]\cdot[u(t)] \qquad (14)$$

Using this similarity the variation of the flux can be expressed using the relation (5) and knowing the initial values. The flux variation will be:

$$\Phi = \Phi_m\left[sin(\omega t + \alpha - \varphi) - sin(\alpha - \varphi)e^{-\frac{R}{L}t}\right] \qquad (15)$$

Where $\Phi_m = \frac{LU}{N\sqrt{R^2+X_L^2}}$ and

$$tg\varphi = \frac{X_L}{R}$$

Using the Simulink model, presented on the figure 6, for the equation (14), the flux variation can be followed.

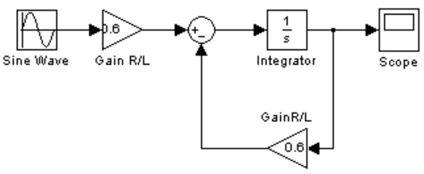

Fig. 6: Simulink model for open circuit transformer functioning on the linear part of the B-H curve

Figure 7 shows the flux variation in transient state. It can be observed that there are two components: an exponential component which is added to the sinusoidal component. So the first amplitude is larger than the flux amplitude in steady state.

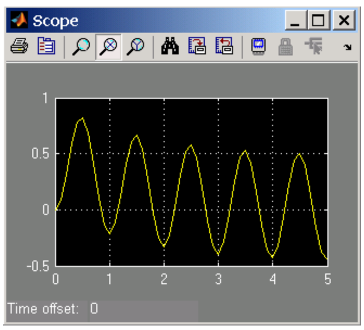

Fig. 7: Flux variation in open circuit transformer

The current variation can be obtained using equation (11). Considering the functioning point at linear part of the magnetization curve; the current variation will have the same form and multiplying with N/L.

If the functioning point is nearly to the saturated part of the magnetization curve, the function *i=f(Φ)* is nonlinear.

The transformer can be studied using an equivalent circuit shown in figure 8. In the equivalent circuit the losses through magnetic leakages, core and windings are not neglected. [6], [8]

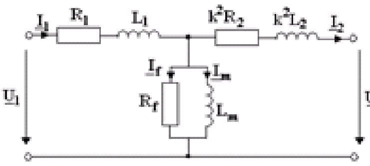

Fig. 8: Equivalent circuit for a real transformer

On the equivalent circuit R_1 and L_1 are the primary winding losses; k^2R_2 and k^2L_2 are the secondary winding losses referred to the primary side; $k = N_1/N_2$; R_f represent the losses in the core and L_m the magnetization and the useful inductivity.

The mathematic model for this equivalent circuit is written using Kirchhoff's laws.

$$U_1 = R_1 I_1 + L_1 \frac{dI_1}{dt} + R_f I_f$$

$$I_1 = I_f + I_m + I_2$$

$$-L_m \frac{dI_m}{dt} + L_2' \frac{dI_2}{dt} + R_2' I_2 + U_2 = 0$$

$$L_m \frac{dI_m}{dt} = R_f I_f$$

$$-R_f I_f + L_2' \frac{dI_2}{dt} + R_2' I_2 + U_2 = 0$$

$$U_1 = R_1 I_1 + L_1 \frac{dI_1}{dt} + R_2' I_2 + L_2' \frac{dI_2}{dt} + U_2 \qquad (16)$$

In these equations $R_2' = k^2 R_2$ and $L_2' = k^2 L_2$. The state parameters will be I_1, I_2, I_m. Two cases can be considered. If no load is connected to the output $I_2=0$ and $U_2 \neq 0$. If a load is connected $\underline{Z_L} = R_L + j\omega L_L$ $I_2 \neq 0$ and $U_2=0$.

Reordering the equations (16) and considering no load to the output the system can be described through:

$$\frac{dI_1}{dt} = \frac{1}{L_1}\left[-(R_1 + R_f) I_1 + R_f I_m\right] + \frac{1}{L_1} U_1$$

$$\frac{dI_m}{dt} = \frac{R_f}{L_m}(I_1 - I_m)$$

$$U_2 = R_f (I_1 - I_m) \qquad (17)$$

The equations (17) can be written in matrix form:

$$\frac{d}{dt}\begin{bmatrix} I_1 \\ I_m \end{bmatrix} = \begin{bmatrix} -\frac{R_1+R_f}{L_1} & \frac{R_f}{L_1} \\ \frac{R_f}{L_m} & -\frac{R_f}{L_m} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_m \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\ 0 \end{bmatrix} \cdot U_1$$

$$U_2 = \begin{bmatrix} R_f & -R_f \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_m \end{bmatrix} \qquad (18)$$

Using Simulink state space simulation model the output of the system is shown on figure 9.

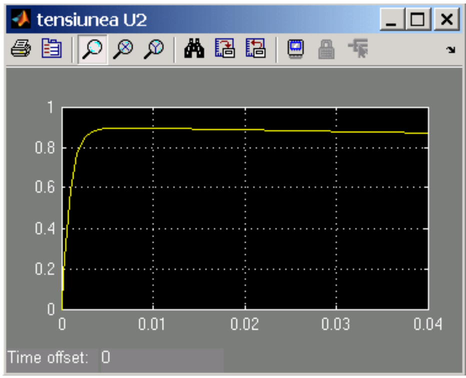

Fig. 9: Output voltage variation

If a Z_L load is connected to the output the current $I_2 \neq 0$, $U_2=0$ and the system is described through:

$$\frac{dI_1}{dt} = \frac{1}{L_1}\left[-(R_1 + R_f) I_1 + R_f I_2 + R_f I_m\right] + \frac{1}{L_1} U_1$$

$$\frac{dI_m}{dt} = \frac{R_f}{L_m}(I_1 - I_2 - I_m)$$

$$\frac{dI_2}{dt} = \frac{1}{L_2' + L_L}\left[R_f I_1 - (R_2 + R_L + R_f) I_2 - R_f I_m\right] \qquad (19)$$

The matrix form of these equations:

$$\frac{d}{dt}\begin{bmatrix} I_1 \\ I_m \\ I_2 \end{bmatrix} = \begin{bmatrix} -\frac{R_1+R_f}{L_1} & \frac{R_f}{L_1} & \frac{R_f}{L_1} \\ \frac{R_f}{L_m} & -\frac{R_f}{L_m} & -\frac{R_f}{L_m} \\ \frac{R_f}{L_t} & -\frac{R_f}{L_t} & -\frac{R_t}{L_t} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_m \\ I_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \end{bmatrix} \cdot U_1 \qquad (20)$$

where $L_t = L_2' + L_L$ and $R_t = R_2 + R_L + R_f$.

The output in this case can be chosing the voltage on the load or the current through the load and secondary winding I_2.

The output I_2 of the system is described through:

$$I_2 = \begin{bmatrix} \frac{R_f}{R_L+R_f} & -\frac{R_f}{R_L+R_f} & 0 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_m \\ I_2 \end{bmatrix} \qquad (21)$$

Using Simulink state space model for simulation the output current variation is presented on figure 10.

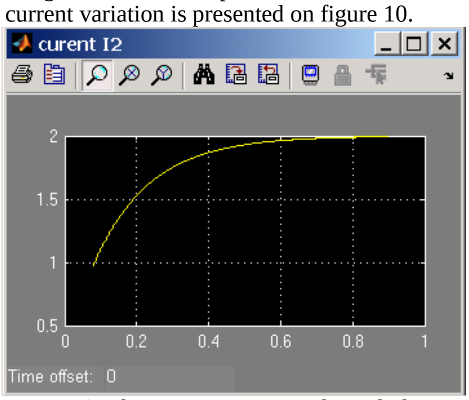

Fig. 10: The current variation through the secondary winding

The delay of the current is explained through magnetization and magnetically coupling between the primary and secondary winding. This secondary current variation influences the primary current through mutual coupling.

5. Conclusions

The transient state appears by any change in a circuit, for example on/off switching. This relative short event of modification of energy in a system influences other devices too. The produced impulses or noises propagate through the net and damage instruments and equipments.

State variables method can be used in transient state analysis. The state parameters were deduced for an RLC circuit and for a real transformer. Using these state parameters the influence of the on/off switching on the current and voltage variation was shown.

At real transformer analysis two cases were discussed. The transformer functioning in open circuit can be compared with an RL circuit. The flux variation in transient state was deduced. Using the equivalent circuit for the real transformer, the losses through magnetic leakages, core and windings are shown. The state parameters deduced in this case include these losses. Using these state parameters the output voltage and the secondary winding's current variation was studied.

Using state parameters to study the transient state can highlight the influence of variation of different components. The state space simulation model makes the study easier and faster.

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