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EXPERIMENTS OF CONTROLERS TUNING BASED ON INTEGRAL CRITERIA

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Abstract

For a system specified by the process, actuator and transducer, the obtaining of a certain operating regime (stationary and transient) is conditioned by the proper choice of a controller's structure and the tuning parameters. In order to satisfy the required performances, in most of the cases, contradictory solutions are reached, finally choosing the compromise solutions that provide acceptable values for all of the quality indices.
Considering the set-point and the disturbance, the PID controllers may be tuned based on a series of criteria, including the relationships that minimize the integral of the error, known as the "minimum error integral tuning". The paper presents the Matlab-Simulink simulation experiments, describing the behavior of a first order plus dead time (FOPDT) system with controllers tuned based on the integral of square error (ISE), the integral of absolute error (IAE) and the integral of time multiplied by absolute error (ITAE).

Keywords: PID controllers, PID tuning, IAE criterion, ITAE criterion

1. Introduction

The proportional – integral – derivative (PID) controllers are the most used in order to control the industrial processes. The obtaining of the imposed performances for the closed-loop system (overshoot, rise time, settling time, peak time and steady state error) requires the optimal tuning of the controller parameters.

The literature highlights experimental methods (e.g. Ziegler-Nichols, Kapelovici etc.) as well as methods based on the minimization of integral squared error. So, a process modeled by first order lag plus time delay, the PID controller designing using the ITAE criterion and a comparative study with the traditional tuning rules are presented in [1]. Fundamentals of process control, PID controllers tuning guide, tools and case studies are very well developed in [2]. In the paper [3], two methods of determining the optimum values of the tuning parameters for the PID controller are presented, being taken into consideration the integral criteria and the square of the error between reference and controlled value. The fundamentals of the IE, ISE, IAE, ITAE, ITSE criteria and the dependences on damping for a second order system are developed in [4]. Tuning methods consist of minimizing the performance indices of the IAE, ISE, ITAE and ITSE criteria combined with genetic algorithm are presented in [5]. The paper [6] examines the performance of ten tuning rules used to compensate six representative processes, involving the ability of the PI and PID controllers to compensate a great number of parameters from industry. For a process modeled by a third order equation, the paper [7] uses different PID tuning formulas, including the minimization of ISE and IAE criteria. The synthesis and analysis of optimal tuning of PID parameters for first order plus time delay, second order plus time delay and second order plus time delay with lead, involving the minimization of the IAE, are developed in [8]. According to [9], the minimizing of the ITAE criterion is also used in order to design a fractional-order PID controller.

The paper [10] presents the time domain optimal tuning of FOPID controller designed for higher order fractional model by using optimization with ITAE criterion. The chapter 12 from [12] presents the methods and relations for controllers tuning, respectively, internal model control, integral of error, on-line controller tuning etc. The chapter 7 from the book [13] is dedicated to the tuning of feedback controllers, including quarter decay ratio response and minimum error integral tuning formulas for set point changes and disturbance input. An analysis of some well-known PID tuning formulas is presented in

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[14].

A novel alternative method to tune an optimal and robust PID controller for open-loop unstable first-order plus time delay systems with gain and phase margins specifications and the IAE and ISE performance criteria of every representative point are proposed by the paper [17].

In this section of the paper a short review of studies on "minimum error integral tuning", from the literature is presented. In subchapter 2 the concepts related to controllers tuning based on integral criteria are presented. In Section 3 Matlab-Simulink experiments of various PI controlled systems, tuned by the Zeigler-Nichols method, ISE, IAE, ITAE criteria respectively are tested and the final considerations are provided in conclusions section.

2. Controllers tuning based on integral criteria

A great number of processes from industry needs to control the parameters as: flow, level, pressure, temperature etc. and are mathematically expressed by the first order plus with dead time (FOPDT) transfer functions [2], [12]-[14]:

$$H_p(s) = \frac{k_p}{T_p s + 1} e^{-\tau s}, \tag{1}$$

where: T_p is the time constant, k_p – amplification; τ - dead time.

To tune the controllers, the criteria use the error defined as the difference between the reference signal, *r(t)* and the output, *y(t)* depending on time:

$$\varepsilon(t) = r(t) - y(t), \tag{2}$$

with the total error, in the $[0,\infty)$ interval, defined as *integral of error* (IE):

$$I_1 = \int_0^\infty \varepsilon(t) dt. \tag{3}$$

The performance indices based on the integral of error are: *integral of absolute error* (IAE), *integral of square error* (ISE), *integral of time multiplied by absolute error* (ITAE), *integral of time multiplied by square error* (ITSE), with the following formulas and time evolution (Fig. 1) [2], [4], [6], [7], [12]-[14]:

$$I_2 = \int_0^\infty |\varepsilon(t)| dt, \tag{4}$$

$$I_3 = \int_0^\infty \varepsilon^2(t) dt, \tag{5}$$

$$I_4 = \int_0^\infty t|\varepsilon(t)| dt, \tag{6}$$

$$I_5 = \int_0^\infty t\varepsilon^2(t) dt. \tag{7}$$

Considering the standard second order system:

$$H_{0II}(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}, \tag{8}$$

the ITAE (I_4) criterion has the biggest variation of the value, depending of the damping value, ξ and is the most selective criterion (Fig. 2).

For the second order system with (ξ, ω_n) parameters, according to the Fig. 2, the values of the I_2, I_3, I_4, I_5 indices have the minimum for the damping $\xi \cong 0.65; 0.5; 0.7; 0.6$.

For a good behavior to the disturbance actions, the PI controllers can be used [2], [4], [6], [13]:

$$H_R(s) = k_R\left(1 + \frac{1}{T_i s}\right), \tag{9}$$

where the tuning parameters (k_R, T_i) are calculated according to (10) and Table 1 [12], [13].

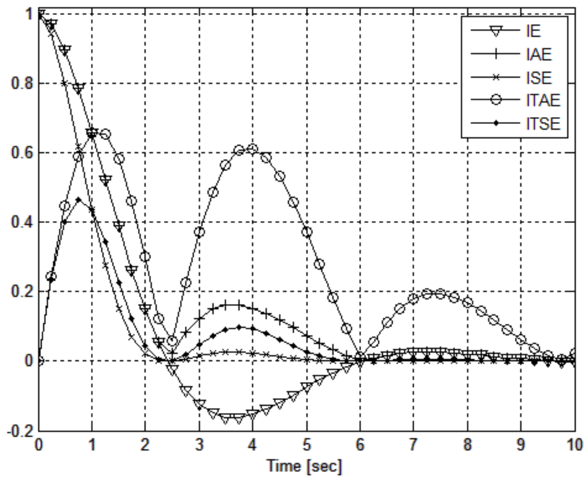

Fig. 1: Time evolution of the integral criteria for $H_0(s) = 1/(s^2 + 0{,}5s + 1)$

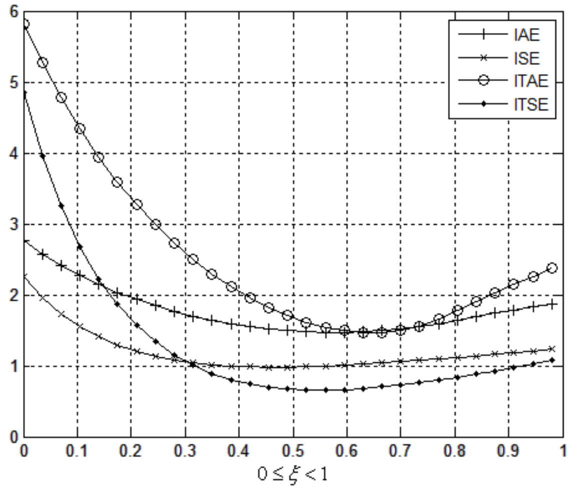

Fig. 2: I_2, I_3, I_4, I_5 criteria variation, depending on ξ, for $H_0(s) = 1/(s^2 + 2\xi s + 1)$

$$k_R = \frac{a}{k_p}\left(\frac{\tau}{T_p}\right)^b ; \; T_i = \frac{T_p}{c}\left(\frac{\tau}{T_p}\right)^d . \qquad (10)$$

Table 1: Coefficient values

	ISE	IAE	ITAE
a	1.279	0.984	0.859
b	-0.945	-0.986	0.977
c	0.535	0.608	0.674
d	0.586	0.707	0.680

For a good behavior to the set point actions, the tuning parameters (k_R, T_i) are calculated according to (11) and Table 2 [13].

$$k_R = \frac{a}{k_p}\left(\frac{\tau}{T_p}\right)^b ; \; T_i = \frac{T_p}{c + d\left(\frac{\tau}{T_p}\right)} . \qquad (11)$$

Table 2: Coefficient values

	ISE	IAE	ITAE
a	0.980	0.758	0.586
b	-0.892	-0.861	-0.916
c	0.690	1.02	1.03
d	-0.155	-0.323	-0.165

3. Simulation experiments

In order to simulate the closed-loop system, the transfer function (1) is used with: $k_p = 0.1$; $T_p = 1.5$ min.; $\tau = 0.25$min.

Considering set point (SP) tracking and disturbance (D) tracking, the tuning parameters for the PI controllers are determined based on the ISE, IAE and ITAE integral criteria. In order to compare the results, a PI controller is determined according to the Ziegler-Nichols (ZN) experimental criterion [2], [7], [12]:

$$k_R = 0.9 \cdot T_p / (\tau \cdot k_p); \; T_i = 3.3 \cdot \tau . \qquad (12)$$

The tuning parameter obtained for the PI controller by the ISE, IAE, ITAE and ZN criteria are shown in Tables 3 and 4.

Table 3: Tuning parameters for PI controller

	SP tracking			
	ISE	IAE	ITAE	ZN
k_R	48.4548	35.4534	30.2471	54
T_i	2.2585	1.5525	1.4963	0.8250

Table 4: Tuning parameters for PI controller

	D tracking			
	ISE	IAE	ITAE	ZN
k_R	69.5382	57.5774	49.4592	54
T_i	0.9812	0.6951	0.6581	0.8250

Fig. 3 and 4 show the step response of the closed-loop system and the disturbance rejection, with PI controller tuned with ZN, ISE, IAE and ITAE criteria, in case of SP tracking [11], [15], [16].

In case of SP tracking, the simulation results highlight the superiority of the ISE, IAE and ITAE methods compared to the ZN method (Table 5).

Table 5: Performances resulted from Fig. 3

	SP tracking			
	ISE	IAE	ITAE	ZN
σ[%]	23	10	5	60
t_t [min]	3	1.5	1.7	3

Fig. 5 and 6 show the step response of the closed-loop system and the disturbance rejection, with PI controller tuned with ZN, ISE, IAE and ITAE criteria, in case of D tracking. In case of D tracking the ITAE method is better than ISE, IAE and ZN methods (Table 6).

Table 6: Performances resulted from Fig. 5

	D tracking			
	ISE	IAE	ITAE	ZN
σ[%]	80	75	64	60
t_t [min]	>4	>3.5	3.5	3.5

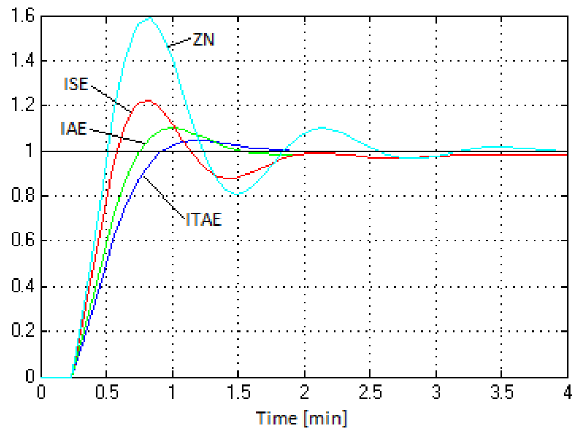

Fig. 3: Step response: SP tracking, SP=1, D=0

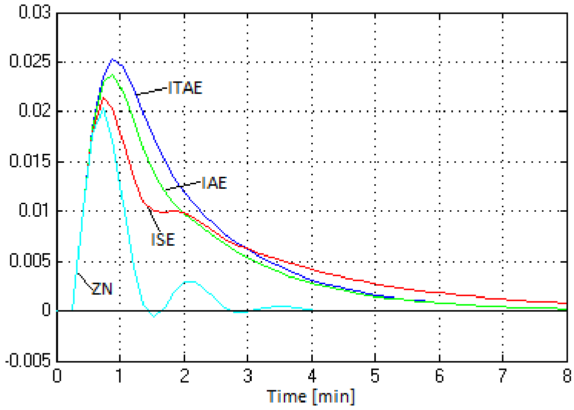

Fig. 4: Step response: SP tracking, SP=0, D=1

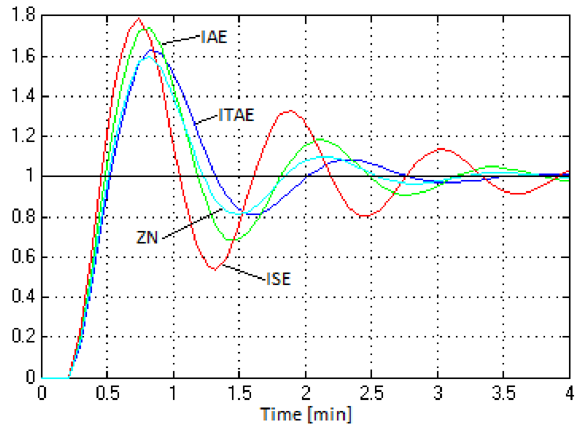

Fig. 5: Step response: D tracking, SP=1, D=0

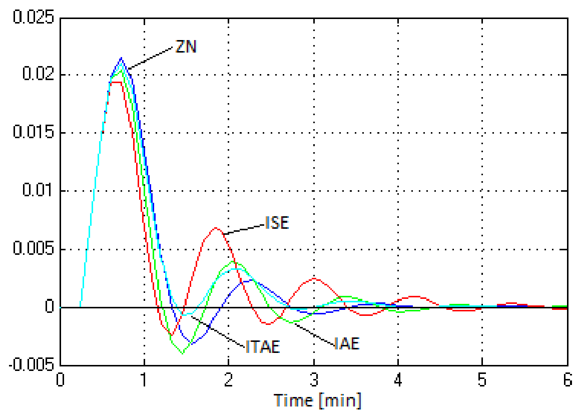

Fig. 6: Step response: D tracking, SP=0, D=1

Fig. 7 presents the evolution of the IAE (I_2), ISE (I_3) and ITAE (I_4) performances indices in case of SP tracking [11], [15], [16]. The simulation scheme in Fig. 8 contains the process with dead time (the actuator and the transducer are included), the PI controller, the reference (SP) and the disturbance (D) signals, and the graphical representations of the output variable and integral of absolute error (IAE) [6], [11], [15], [16].

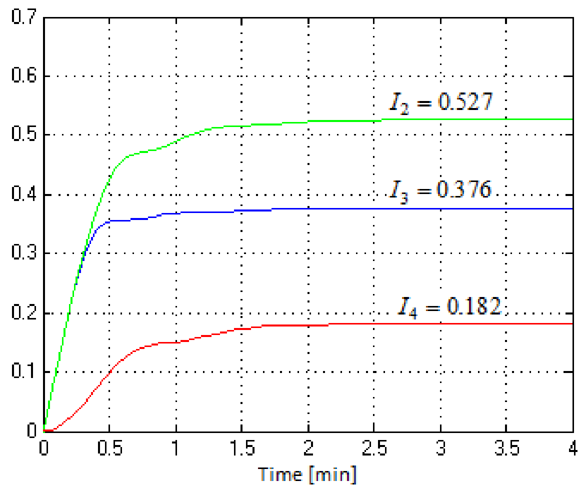

Fig. 7: IAE (I_2), ISE (I_3) and ITAE (I_4) indices: SP=1, D=0

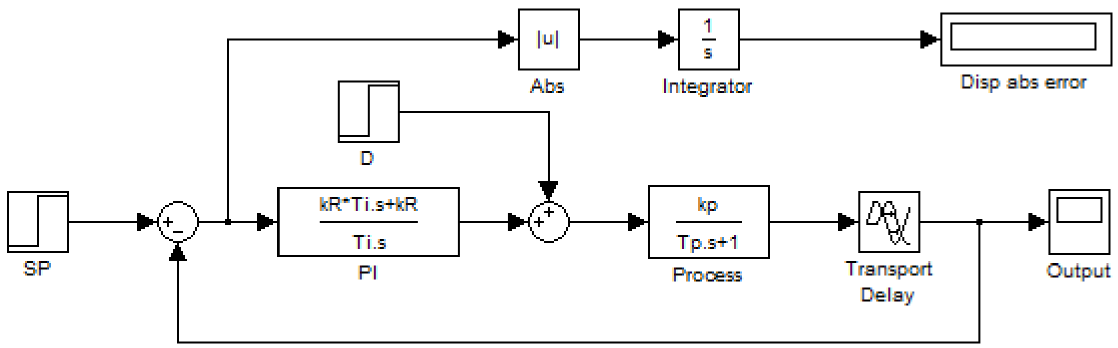

Fig. 8: The simulation scheme

4. Conclusions

The tuning of a PID controller is the most important task that needs to be solved in order to achieve the set-up performances imposed in the closed-loop system: steady state error (ε_{st}), overshoot (σ) and settling time (t_t).

In this paper, the author presents a comparative study regarding the different tuning methods applied to the PI controller used for a FOPDT system.

The tuning methods used in the paper are based on the minimizing of the ISE, IAE and ITAE criteria. The obtained performances are compared to the performances obtained using the PI controller, tuned by the ZN experimental criterion.

The ZN criterion provides a faster disturbance rejection than the ISE, IAE and ITAE criteria (Fig. 4), considering the PI controllers tuned for SP tracking.

On the other hand, the PI controller tuned by ZN criterion and the PI controller tuned by the ISE, IAE and ITAE criteria, in case of D tracking (Fig. 6), provide identical performances.

The criterion (3) I_1, integral of error (IE), is not relevant for the underdamped responses $(0 < \xi < 1)$, due to the mutual cancellation of areas with positive and negative errors (Fig. 1). However, the criterion (3) can be used for the cases where the damping $\xi \geq 1$ (critically damped, overdamped).

According to the Fig. 2 the ITAE (I_4) criterion is the most selective, having the biggest variation (depending of the damping), and assures minimum values of the performances indices (Fig. 7).

Due to the integral component of the PI controller, the steady state error, $\varepsilon_{st} \cong 0\%$, in all the cases.

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