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INSTANTANEOUS FREQUENCY ESTIMATION OF DISCRETE TIME SIGNALS

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Abstract

The classical concept of instantaneous frequency, obtained by differentiating the instantaneous phase is one of the most used approaches. Nonlinear signals usually have nonlinear and nonstationary behavior. Revealing hidden properties of time discrete signals could be important in understanding specific phenomena or processes. This paper uses simulated signals to prove the utility of instantaneous frequency estimation in dedicated signal processing. The procedure is based on empirical mode decomposition of the signal into monocomponents. The Hilbert transform of these monocomponents reveals they instantaneous frequencies. There are certain mathematical requirements and limitations for signals that the proposed procedure could perform proper instantaneous frequency estimation. The used signals are artificial and the procedure is carried out in MATLAB.

Key words: instantaneous frequency, empirical mode decomposition, Hilbert transform

1. Introduction

Discrete time signal analysis is a very important task in research and practical experimentations. Usually signal analysis defines the parameters needed to construct and represent the model of a studied phenomenon. In most of cases the obtained data in form of discrete time signals are non-stationary, nonlinear and noisy. This paper focuses on a new nonlinear signal processing method in time-frequency domain named instantaneous frequency estimation. The instantaneous frequency generated many controversial discussions about its non-unique way definitions but remains an extremely useful analyzing tool in certain conditions [1], [2]. The most important fields where instantaneous frequency estimation is used are the exploration seismology, electrical engineering, biomedical applications [3]. This paper is organized as follows. The second chapter presents the theoretical background of the used method; chapter three presents the proposed procedure and the fourth the obtained results. Concluding remarks are done in the fifth chapter.

2. Estimation of instantaneous frequency

There are many elaborated and well-established methods available for processing nonstationary data. Since most of the methods still depend on time-frequency analysis, they are limited to linear systems only and usual use predefined analyzing functions. In addition, priori knowledge about the signal is required in order to adapt the analyzing tool. Lately, new and adaptive methods were elaborated, which are mostly signal driven and no require expansion functions. One of these methods is the Empirical Mode Decomposition (EMD) presented in this work. While the concept of frequency is very old, the concept of instantaneous frequency (IF) is relatively new, originating with Nobel Laureate, Dennis Gabor (1946) [7]. The classical instantaneous phase originates with phase modulation concepts [4], in which the signal, f(t), has a representation of the form,

$$f(t) = A(t) \cdot e^{j\omega_0 t + j\int_0^t m(\tau)d\tau} \qquad (1)$$

Where the amplitude A(t) is a very slowly-varying function of time, with ω_0 the carrier frequency, and with $m(\tau)$ a slowly-varying modulation function. The signal looks very similar to the basic exponential,

$$f(t) = e^{j\omega_0 t} = \cos(\omega_0 t) + j\sin(\omega_0 t) \qquad (2)$$

A real signal is one that exhibits Hermitian symmetry between the positive-frequency and negative-frequency components [6]. The negative-frequency components of a real signal may be eliminated from the signal representation without losing information by forming the analytic signal given by

$$S(t) = f(t) + j\mathrm{H}[f(t)] = f(t) + jg(t) \quad (3)$$

where g(t) is obtained by the Hilbert transform (HT), a convolution operation defined by

$$g(t) = \int_{-\infty}^{+\infty} f(\tau)\frac{1}{t-\tau}d\tau \quad (4)$$

A signal is analytic with a real DC component if its imaginary part is the HT of its real part [5]. Basically, the Hilbert transform maps cosines into sines and sines into negative cosines. Thus, each Fourier transform component is phase rotated by $\pi/2$, with positive frequency components phase-delayed by $\pi/2$ and negative frequency components phase-advanced by $\pi/2$. In the case of a real low-pass signal, removal of negative frequencies reduces the total bandwidth, allowing the signal to be sampled at half the usual Nyquist rate without aliasing [13,14]. and avoids the appearance of some interference terms generated by the interaction of positive and negative components in quadratic time-frequency decompositions [9].

A monocomponent signal as S(t) has an analytic associate of the form (2), any complex signal would qualify as monocomponent because any complex signal can be written in this form [11]. A multicomponent signal may be described as the sum of monocomponent signals [17]. Because the IF of a signal indicates the dominant frequency of the signal at a given time, the requirement of a monocomponent signal is to have IF a single-valued function of time. So, it is needed a decomposition into mono-components but the procedure is not necessarily unique [12]. One of them is the Empirical Mode Decomposition (EMD) which fragments any signal in intrinsic mode functions (IMFs) as follows [17]:

For a given discrete signal s(t), m_1 is the mean value of its upper and lower envelope curves defined by local maxima and minima. The first prototype component c_1 is computed:

$$c_1 = s(t) - m_1 \quad (5)$$

In the second sifting process, c_1 is treated as the data, and m_{11} is the mean of c_1's upper and lower envelopes:

$$c_{11} = c_1 - m_{11} \quad (6)$$

This sifting procedure is repeated k times, until c_{1k} is an IMF, that is:

$$c_{1(k-1)} - m_{1k} = c_{1k} \rightarrow c_1 \quad (7)$$

If this component satisfies the stop criteria for IMF sifting, c_1 this will be the first IMF. The residual signal will be constructed as follows:

$$r(t) = s(t) - c_1 \quad (8)$$

If r(t) satisfies the stop criterion for EMD then it will be the final residual signal and the EMD process will be finished. [16]. An IMF is defined as any function having the same (or differing at most by one) numbers of zero-crossing and extrema, and also having symmetric envelopes defined by the local maxima and minima, respectively. With the Hilbert transform, the IMF's yield instantaneous frequencies as functions of time.

3. The proposed procedure

The analyzed signal is decomposed in Intrinsic Mode Functions (IMFs) using the already mentioned sifting procedure. The obtained IMFs are Hilbert transformed in order to obtain analytic signals which instantaneous frequencies are estimated through the presented algorithm. The principle of the proposed procedure is presented on figure 2.

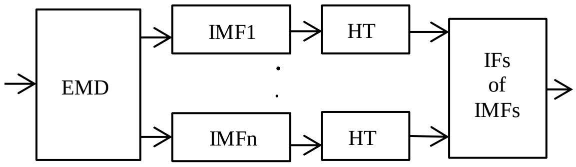

Fig. 1: The proposed procedure

For different properties different IMFs should be studied, usually the frequency of IMF decreases with the decomposition order.

For higher frequency related behavior the first IMFs must be evaluated, for slow changes the higher order IMFs could be responsible.

4. Simulation results

The used signals are created artificially; the whole estimation procedure was carried out under MATLAB environment [10].

The types and the lengths of signals were chosen in order to have enough gained information to be able to make concluding remarks.

The first signal used is a ‘quadchirp’ from Matlab test signals, the signal and the resulted IMFs are presented on figure 2

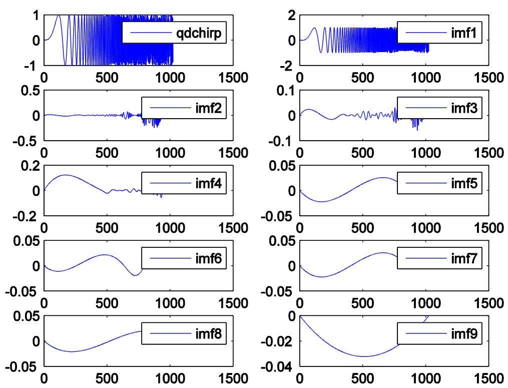

Fig. 2: The resulted IMFs of quadchirp signal

The decomposition through the sifting method is very accurate, the reconstruction error (the difference between the original signal and the sum of resulted intrinsic functions) has a very small value. It is important to mention that the decomposition process depends on signal length, therefore the number of obtained IMFs is strongly dependent on signal properties. The second signal is the so called 'mishmash' signal (also from Matlab) presented on figure 4. The reconstructed signals and the approximation errors for the two test signals are presented on figure 3 and 4.

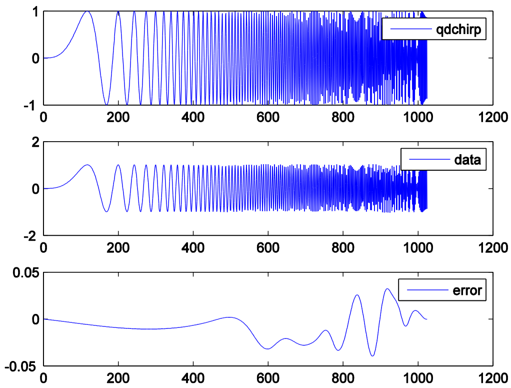

Fig. 3: The approximation error of EMD ('quadchirp')

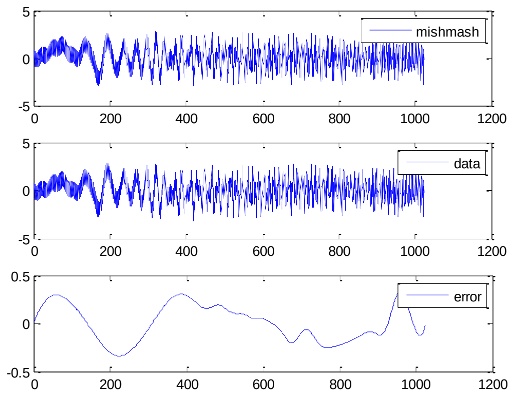

Fig. 4: The approximation error of EMD ('mishmash')

The estimated instantaneous frequencies of obtained monocomponents carry time localized information about frequency domain behavior.

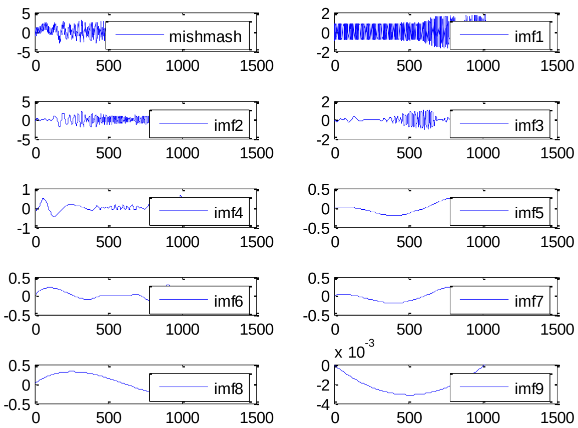

Fig. 5: The EMD of 'mishmash'

As presented on figure 5, the higher order IMFs contain the lower frequency parts. It is convenient to choose the corresponding IMF in order to search for specific signal changes if the frequency domain is already known. Instantaneous frequency (IF) express the signal spectral variations as a function of time. This can be used to detect unwanted changes in the analyzed signal. The instantaneous frequencies obtained for the first six component IMFs are presented on figure 6.

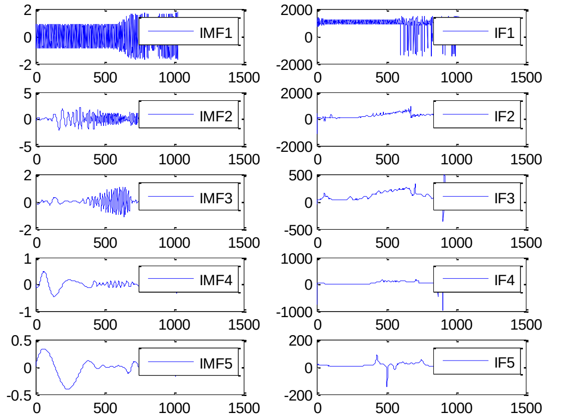

Fig. 6: The Ifs for first six IMFs of 'mishmash' signal

The Hilbert transform based instantaneous frequency estimation gave the following results for the 'quadchirp' signal. At first on figure 7 the decomposition in nine IMFs is presented.

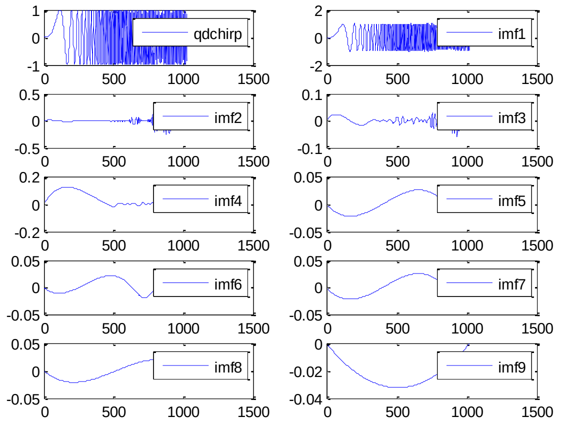

Fig. 7: The IMF/IF pairs ('leleccum' signal) obtained

The obtained instantaneous frequencies for the first six IMFs are presented on figure 8.

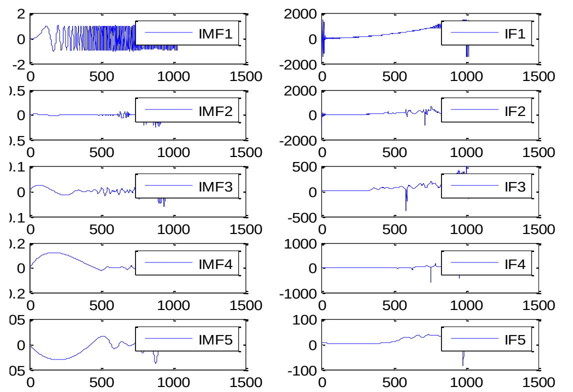

Fig. 8: The IMF/IF pairs obtained for the 'quadchirp' signal

tutorial review, Digital Signal Processing, vol. 40, pp.1-30.

The linear variation of the signal frequency can be easily observed on the IF of the first IMF, it is a specific feature for this kind of signal.

5. Conclusions

The instantaneous frequency (IF) is a basic parameter which may be used to describe the nonstationarity in a process or a signal. The EMD procedure can be viewed as a generalized time-frequency decomposition in monocomponents without expansion functions. The analytic associate of a monocomponent asymptotic signal is fully characterized by its instantaneous amplitude and instantaneous phase, from which we can determine its IF. Time-frequency methods are preferred for a wide range of applications in which the signals have time-varying spectral characteristics or multiple components for which the variables time and frequency are related as in biomedical signal analysis, fault detection and parameter estimation. Future work will focus on using linear and nonlinear filter banks to obtain IMFs and perform the Hilbert transform.

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